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HEAT TRANSFER ANALYSIS FOR UNSTEADY HIGH VELOCITY PIPE FLOW

PART I. ON MINIMIZATION OF TEMPERATURE DISTORTION IN THE THERMOCOUPLE CAVITY. PART II. IMPROVED ACCURACY IN THE PREDICTION OF SURFACE HEAT FLUX AND TEMPERATURE BY INTRINSIC THERMOCOUPLE. PART III. PREDICTION OF TRANSIENT SURFACE HEAT FLUX AND TEMPERATURE ON A HOLLOW CYLINDER

IOWA INSTITUTE OF HYDRAULIC RESEARCH
IOWA CITY, IOWA

OCTOBER 1976

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Part II Improved Accuracy in the Prediction of Surface Heat Flux and Temperature by Intrinsic Thermocouple

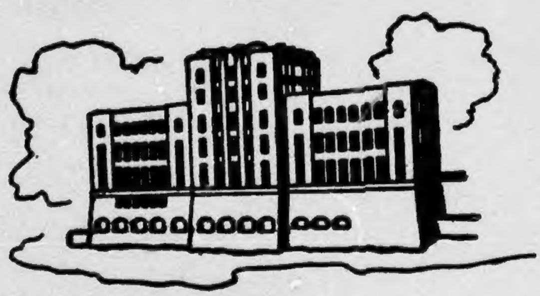
Part III Prediction of Transient Surface Heat Flux and Temperature on a Hollow Cylinder

by

Ching Jen Chen, Jenq Shing Chiou, Peter Li
and Hsai Yin Lee

Sponsored by

U.S. Army Research Office
Grant Number DAA-G29-76-G-0123



IIHR Report No. 195

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Iowa City, Iowa

October 1976

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ABSTRACT

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ACKNOWLEDGEMENTS

The investigators would like to thank Mr. D.M. Thomsen of General Rodman Laboratory, Rock Island Arsenal, Rock Island, Illinois for his constant participation in the course of the research.

TABLE OF CONTENTS

PART I ON MINIMIZATION OF TEMPERATURE DISTORTION IN THE THERMOCOUPLE CAVITY

I. INTRODUCTION

A direct measurement of transient surface temperature and heat flux is often difficult. For example, a surface involves two modes of heat transfer, say, radiative and convective heat transfer. In this case if the measuring probe has a different radiative property than that of the surface, erroneous measurements will result. A piston or projectile sliding over the cylindrical surface is another case where the direct measurement at the surface is difficult. A surface involving melting or ablation is also difficult to make direct measurement. Therefore, indirect estimation by inverting the temperature history inside the heat conducting solid as measured by a thermocouple is often used for prediction of the surface temperature and heat flux. Beck [1], Hernning and Parker [2], Frank [3], Imber and Khan [4], Stolz [5], Chen and Thomsen [6] have developed different inversion solutions for this purpose. All of these solutions assumed that the cavity drilled into the solid does not distort the true temperature distribution. Thus, it is important that the temperature measurement by an interior probe is accurate and involves least distortion or error. Theoretically Beck [7], Masters and Stein [8], Burnett [9], and Chen and Li [10] studied the distortion of the temperature field in the present of thermocouple and its cavity. Experimentally Chen and Danh [11] showed that appreciable distortion, say 10%, of temperature field may exist for a normal implant of the thermocouple into a solid body. From studies of Chen and Li [10] and Beck [7] they found that with a proper combination of the thermocouple cavity diameter, its depth, and the thermocouple material and its diameter

the distortion of temperature field with respect to space or time can be minimized if not eliminated. In this report we study the optimum combination of these parameters such that at a given situation one knows what is the best combination to use and what is the magnitude of the temperature distortion.

II. FORMULATION OF PROBLEM

In the present study we consider a disk depicted in Figure 1 which has a thickness D and is drilled a cavity of a diameter to a depth of ε distance from the heating surface. The heat flux Q is assumed to be constant and the upper surface of the disk is assumed to be insulated. A thermocouple of a diameter d_t is then welded on the cavity base. Furthermore, the disk may be thought to approximate a cut out from a hollow cylinder if the radius of the disk is small compared with the radius of the cylinder. The diameter of the disk is choosen to be 2D outside which the temperature distortion due to the thermocouple cavity becomes negligiable. For this to be true one needs to restrict the ratio of cavity diameter to the disk diameter d/2D be small. The unfilled cavity can be air or insulating material.

The basic idea to minimize or to eliminate the temperature distortion is based on a proper choice of the thermocouple size and the material which has a higher thermal conductivity than that of the disk so as to conduct more heat away at the cavity base balancing the insulation effect of the insulator in the cavity.

Let X and Y be respectively the coordinate along and normal to the heated disk surface and the Y axis coincide with the axis of the cavity.

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Let X and Y be respectively the coordinate along and normal to the heated disk surface and the Y axis coincide with the axis of the cavity.

The thermal conductivity is assumed to be constant and the temperature distribution is axisymmetrical. The governing equations for the transient heat conduction in dimensionless form are:

for the disk (subscript "1", see Figure 2)

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial y^2} \tag{1}$$

for the insulating material in the cavity (subscript "2")

$$\frac{\partial\theta}{\partial\tau} = \frac{\alpha_2}{\alpha_1}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{2}$$

for the thermocouple (subscript "3")

$$\frac{\partial\theta}{\partial\tau} = \frac{\alpha_3}{\alpha_1}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{3}$$

where $\tau = \alpha_1 t/D^2$ is the dimensionless time, $x = X/D$ the dimensionless radial coordinate, $y = Y/D$, the dimensionless distance normal to the heated surface. α_1, α_2, and α_3 are respectively the thermal diffusivity of the disk, insulating material, and the thermocouple. The dimensionless temperature θ is defined as $T\kappa_1/QD$ where T is the temperature above the initial, uniform temperature.

The intial temperature of the disk is then

$$\theta\ (x, y, 0) = 0 \tag{4}$$

The boundary conditions, see Figure 2, are:

a constant heat flux at the lower surface,

$$y = 0 \qquad \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 1 \tag{5}$$

the insulation at the upper surface,

$$y = 0 \qquad \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 0 \tag{6}$$

the zero temperature distortion at the edge of the disk,

$$x = 1 \qquad \left.\frac{\partial\theta}{\partial x}\right|_{x=1} = 0 \tag{7}$$

and the axisymmetric condition at the cavity axis

$$x = 0 \qquad \left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0 \tag{8}$$

The condition (7) of the zero temperature distortion was verified by Chen and Li [10] in their early calculation when the cavity diameter is one tenth of the disk diameter. In addition to the above boundary condition the temperature and heat flux at the interface of the disk, thermocouple, and insulating material are taken to be continuous.

III ANALYSIS

There are five parameters that can be varied for the present analysis. They are (a) the dimensionless distance from the base of the cavity to the heated surface ε/D, (b) the size of the cavity d/D, (c) the ratio of the thermocouple diameter to that of the cavity, d_t/d.

(d) the thermal conductivity ratio κ_2/κ_1, and κ_3/κ_1 which comes from the continuity of heat flux at interfaces. (e) the ratio of the product of density and specific heat $\rho_3 c_3/\rho_1 c_1$ (or equivalently to the ratio of of thermal diffisivity α_3/α_2 or α_2/α_1). The subscripts 1, 2 and 3 denote the disk, insulation and thermocouple materials.

Because of the complexity of the geometry and the multicity of material the method of finite element technique as discussed by Wilson and Nikel [12] is adapted with the aid of a computer program developed by Wilson [13]. The present problem is subdivided into finite element as required by the method, see Figure 2. The dimensionless pie section is subdivided into 121 finite elements with 12 dividing lines on both coordinates. Each element is defined by four nodal points where nodal points are denoted by intersections of the dividing lines and numbered as shown in Figure 2. The material property corresponding to each element is then assigned to the program developed by Wilson [13] The solution at each nodal with respect to time is then obtained.

For numerical calculation three typical values of the distance from heated surface to the base of the cavity ε/D are chosen to be 0.04, 0.1 and 0.2. The cavity diameter is fixed at one tenth of the disk diameter. The thermocouple to cavity diameter ratio d_t/d is made to vary 0, 0.2, 0.4, 0.6, 0.8 and 1.0. Regarding the range of the ratio of the thermal conductivity and the ratio of the product of density and specific heat we surveyed these ratios for the commonly used thermocouples in Table 1 [14] and plotted in Figure 3. Figure 3b shows the ratio of

thermocouple conductivity to that of the disk κ_3/κ_1 versus the ratio of density - specific heat product $\rho_3 c_3/\rho_1 c_1$ where the value of the conductivity is taken to be the average value between 200 and 800°K. One sees that for most practical situations the ratio of density - specific heat product $\rho_3 c_3/\rho_1 c_1$ is approximately constant at 0.7 except when the conductivity ratio κ_3/κ_1 is small. Thus the value of $\rho_3 c_3/\rho_1 c_1$ and κ_3/κ_1 for calculation are chosen, as shown in triangular symbols of Figure 3b, to cover the practical range. The corresponding value for $\rho_2 c_2/\rho_1 c_1$ and κ_2/κ_1 for the insulation material are chosen to be fixed at 0.5 and 0.005 which is a typical value for Teflon insulating material and is also approximately the order of magnitude for air.

IV. RESULTS AND DISCUSSIONS

Numerical results of the calculations are presented in Tables 2 to 4 and Figures 4 to 9. The percentage error of temperature is defined as the distorted temperature divided by a reference temperature defined by QD/κ_1. Tables 2 to 4 give the percentage error of temperature distortion) as a function of time for different values of the parameters d_t/d (0 to 1.0), κ_3/κ_1 (0.5 to 10), $\rho_3 c_3/\rho_1 c_1$ (0.5 to 1.8) and ε/D (0.02 to 0.1).

Figures 4, 5 and 6 show the three typical temperature distributions in the steel disk near the thermocouple junction for the case $\varepsilon/D = 0.06$ $d/2D = 0.1$ at the time $\tau = 0.08$. Figure 4 is the temperature distribution when the cavity is filled entirely with the insulation material (Teflon "2" $\kappa_2/\kappa_1 = 0.005$, $\rho_2 c_2/\rho_1 c_1 = 0.5$). The dimensionless isotherms

$T\kappa_1/QD$ shows the distortion of temperature distribution. One sees that the insulation effect on the heat transfer near the cavity base not only creates a much higher junction temperature of $T\kappa_1/QD = 0.342$ than the undistorted one of 0.255 at the edge of the disk giving an error of 8.7% but also causes a hot spot at the heating surface with a higher temperature of $T\kappa_1/QD = 0.374$ over the undistorted one of 0.311. On the other hand the temperature distribution in the insulation material is much lower than the true temperature creating a large temperature gradient at the base of the cavity. Figure 5, contrary to Figure 4, is the temperature distribution when the cavity is completely filled with thermocouple material whose thermal conductivity is ten times larger than the disk material e.g., copper versus steel. Now over-conduction of heat by the thermocouple has created a cold spot at the base of the cavity giving, $T\kappa_1/QD$ of 0.17 versus the undistorted one of 0.255 with an error of 8.5% as well as at the heating surface with $T\kappa_1/QD$ of 0.236 versus 0.311. The temperature distribution in the thermocouple now becomes higher than the undistorted one in the disk. By properly choosing the ratio of thermocouple diameter to that of the cavity one may minimize these distortions of the temperature response at the base of the cavity. This is shown in Figure 6 where the thermocouple diameter d_t is choosen to be 0.4 of the cavity diameter d with $\kappa_3/\kappa_1 = 10$, e.g. copper-steel combination. Figure 6 shows that distortions at the base of the cavity and at the heating surface are almost eliminated giving the temperature $T\kappa_1/QD$ of 0.245 and 0.320 with respectively the error of 1% and 0.9%.

In order to examine the details of the distorted temperature response at the base of the cavity we tabulated the results in Tables 2, 3 and 4 and plotted the error percentage as function of the ratio of the thermocouple diameter to that of the cavity for different cavity depth, time, and thermal conductivity in Figures 7, 8 and 9. In these Figures the ρc ratio ranges from 0.7 to 1.3 which covers most of the practical application.

In the case when the ρc ratio is equal to or less than one the error in temperature response at the base of the cavity in these figures are all positive or overheat for $\kappa_3/\kappa_1 \leq 1$. This is because the thermal conductivity of the thermocouple is less than that of the disk and the heat capacity ρc of the thermocouple is also small. Therefore, no extra conduction of heat can be achieved by the thermocouple to compensate for the blocking of the heat transfer by the insulation material in the cavity. On the other hand, if $\kappa_3/\kappa_1 > 1$ the error of temperature varies from positive value for $d_t/d = 0$ to some negative value as d_t/d approaches 1. Thus for $\kappa_3/\kappa_1 > 1$ a properly chosen combination of thermocouple and insulation material can minimize the error. For example, in Figure 7 a combination of $\kappa_3/\kappa_1 = 10$, $\rho_3 c_3/\rho_1 c_1 = 0.75$, $\varepsilon/D = 0.1$ and $d_t/d = 0.5$ produces almost negligible error. This combination shown to be optimum at $\tau = 1.0$ in Figure 7 is also optimum for other time periods (see Table 2). Therefore once an optimum combination of parameters is chosen it is valid throughout the entire transient period of an experiment. From Figures 7, 8 and 9 one can also see that the optimum ratio of d_t/d which gives zero temperature error decreases as the κ_3/κ_1 ratio increases.

This implies that for the thermocouple with a larger thermal conductivity a smaller diameter is sufficient to eliminate the temperature distortion. The result shown in Figures 7, 8 and 9 can in general be adapted for use in practical application to choose the size of thermocouple and cavity, the thermocouple material and the depth of the cavity to be drilled.

As mentioned earlier that when $\kappa_3/\kappa_1 \leq 1$ and $\rho_3c_3/\rho_1c_1 \leq 1$ the error of the temperature response at the base of the cavity are all positive. However we found (see Tables 2.2, 3.2 and 4.2) that if ρ_3c_3/ρ_1c_1 ratio is made large enough during the transient period the error of temperature response at the base of the cavity may indeed become negative even when $\kappa_3/\kappa_1 \leq 1$. Physically although the thermal conductivity of the thermocouple κ_3 is smaller than that of the disk material, but with a larger heat capicitance $\rho_3c_3/\rho_1c_1 > 1$ the thermocouple is still capable of absorbing extra heat flux and hence eliminates the temperature distortion at the cavity base during the transient period. To illustrate this fact we examine Figure 7 (or see Table 4.2) for the data of $\kappa_3/\kappa_1 = 1$ and $\rho_3c_3/\rho_1c_1 = 1.3$. One sees that when $d_t/d = 1$ the temperature distortion can indeed be negative. Therefore, if d_t/d are choosen between 0.8 and 1 the error can be minimized. However one must keep in mind that the elimination of error by heat capacitance can work during the transient period only for once a steady state conduction is established the heat capacity ρc will no longer have any effect and over heating at the cavity base eventually will develop. This can be seen best from the governing equation (1) that at steady state the unsteady term which contains ρc product is zero and is not a parameter affecting the distortion.

Another important fact that should be mentioned is that in general the optimum choice of d_t/d ratio for given κ_3/κ_1 and ρc ratio does not vary very much with the variation of ε/D ratio. The insensitivity of the optimum d_t/d ratio to the ε/D ratio ranging from 0.02 to 0.1 means that the distortion of the temperature is insensitive to the cavity depth or the thickness of the disk. This fact was already pointed out by Chen and Danh [11] in their experiment that the temperature distortion at the base of the cavity is more sensitive to the variation of the cavity diameter than the depth of the cavity drilled.

As an example of a practical application, let us consider a measurement of the transient temperature response of an engine block made of aluminum. From Figure 3b we know that aluminum has high thermal conductivity. Therefore copper-constantan thermalcouple which has a higher thermal conductivity than aluminum should be chosen. For this material combination we have $\kappa_3/\kappa_1 = 1.69$ $\rho_3 c_3/\rho_1 c_1 = 1.3$. Now if the thermocouple cavity is drilled such that $\varepsilon/D = 0.1$ then from Figure 7 interpolating between $\kappa_3/\kappa_1 = 2$ and 1 for $\rho_3 c_3/\rho_1 c_1 = 1.3$ we find that the optimum d_t/d for $\kappa_3/\kappa_1 = 1.69$ is approximately 0.7

One disadvantage of invoking finite element analysis is that the result does not give a clear functional relation among the parameters involved. In an attempt to obtain a simple and useful relation to relate the various parameters we note the following fact and result: (a) the optimum d_t/d ratio for zero temperature distortion is a strong function of κ_3/κ_1 and ρc ratio but is relatively insensitive to the ε/D ratio, (b) from the theoretical reasoning the d_t/d ratio is independent

of ρc ratio if the problem is steady state. A simple steady one dimensional analysis in which the thermocouple and the insulation material in the cavity is made to conduct the same amount of heat that would be transferred without the cavity gives the relation

$$d_t/d = \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \tag{9}$$

Using the above equation as a base we find that for the transient heat conduction as calculated by the finite element method the following equation (10) correlates very well with the optimum d_t/d ratio.

$$d_t/d = (\rho_3 c_3/\rho_1 c_1)^{0.3} \sqrt{(\kappa_1 - \kappa_2)/(\kappa_3 - \kappa_2)} \tag{10}$$

Equation (10) gives an error or distortion of no more than two percentage points. In practice equation (10) may be used as a rule of thumb.

V. CONCLUSION

An analysis of the temperature distortion caused by the cavity drilled into a disk to accommodate the thermocouple has been studied. The calculation is carried out for the case of constant heat flux. It is shown that the temperature at the base of the cavity distorted from that without a cavity can be eliminated by a properly chosen combination of the ratio of the thermocouple diameter to the cavity diameter, d_t/d and the thermocouple material κ_3/κ_1. The optimum ratio of d_t/d can be found from Figures 7, 8 and 9 or Tables 2, 3 and 4, or approximately from equation (10). As a rule the thermocouple must be chosen

to have a higher thermal conductivity than that of the heat conducting solid. The cavity diameter should be as small as practically possible. For the case of time dependent surface heat flux the present result may be also used as a general guide.

REFERENCES

(1) Beck, J. V., "Nonlinear Estimation Applied to the Nonlinear Inverse Heat Conduction Problem", International Journal of Heat and Mass Transfer, Vol. 13, 1970 p. 703-716.

(2) Herring, C. D., and Parker, R., "Transient Response of an Intrinsic Thermocouple", Journal of Heat Transfer, Trans. ASME, Series C, Vol. 39, 1967, p. 146.

(3) Frank, I., "An Application of Least Square Method to the Solution of Inverse Problem of Heat Conduction", Journal of Heat Transfer, Vol. 85, No. 4, 1963, p. 378-379.

(4) Imber, M., and Khan, J., "Prediction of Transient Temperature Distributions with Embedded Thermocouple", Journal of AIAA, Vol. 10, No. 6, 1972, p. 784-789.

(5) Stolz, G. Jr., "Numerical Solutions to an Inverse Problem of Heat Conduction for Simple Shapes", Journal of Heat Transfer, Vol. 82, 1960, p. 20-26.

(6) Chen, C. J., and Thomsen, D. M., "On Transient Cylindrical Surface Heat Flux Predicted from Interior Temperature Response.", AIAA Journal, Vol. 13, No. 5, May 1975, p. 697-699.

(7) Beck, J. V., "Thermocouple Temperature Disturbances in Low Conductivity Materials", Transaction of the ASME, May 1962, p. 124-131.

(8) Masters, J. I., and Stein, S., "Effect of an Axial Cavity on the Temperature History of a Surface Heated Slab", The Review of Scientific Instruments, Vol. 27, No. 12, 1956, p. 1065-1069.

(9) Burnett, D. R., "Transient Temperature Measurement Errors in Heated Slabs for Thermocouples Located at the Insulated Surface", Journal of Heat Transfer, Vol. 84, No. 4, Nov. 1961, p. 505-506.

(10) Chen, C. J., and Li, P., "Error Analysis of an Intrinsic Transient Heat Flux Sensor", to be presented at the 16th National Heat Transfer Conference, St. Louis, Missouri, August 8-11, 1976.

(11) Chen, C. J., and Danh, T. M., "Transient Temperature Distortion in a Slab Due to Thermocouple Cavity", AIAA Journal, Vol. 14, 1976.

(12) Wilson, E. L., and Nickel, R. E., "Application of the Finite Element Method to Heat Conduction Analysis", Journal of Nuclear Engineering and Design, Vol. 4, 1966, p. 276-286.

(13) Wilson, E. L., "Transient Temperature Analysis of Plane and Axisymmetric Solids", Computer Programming Series, University of California, Berkeley, August 1965.

(14) Omega Eng. Inc., "Temperature Measurement Handbook", Box 4047, Springdale Station, Stamford, Conn. 06907.

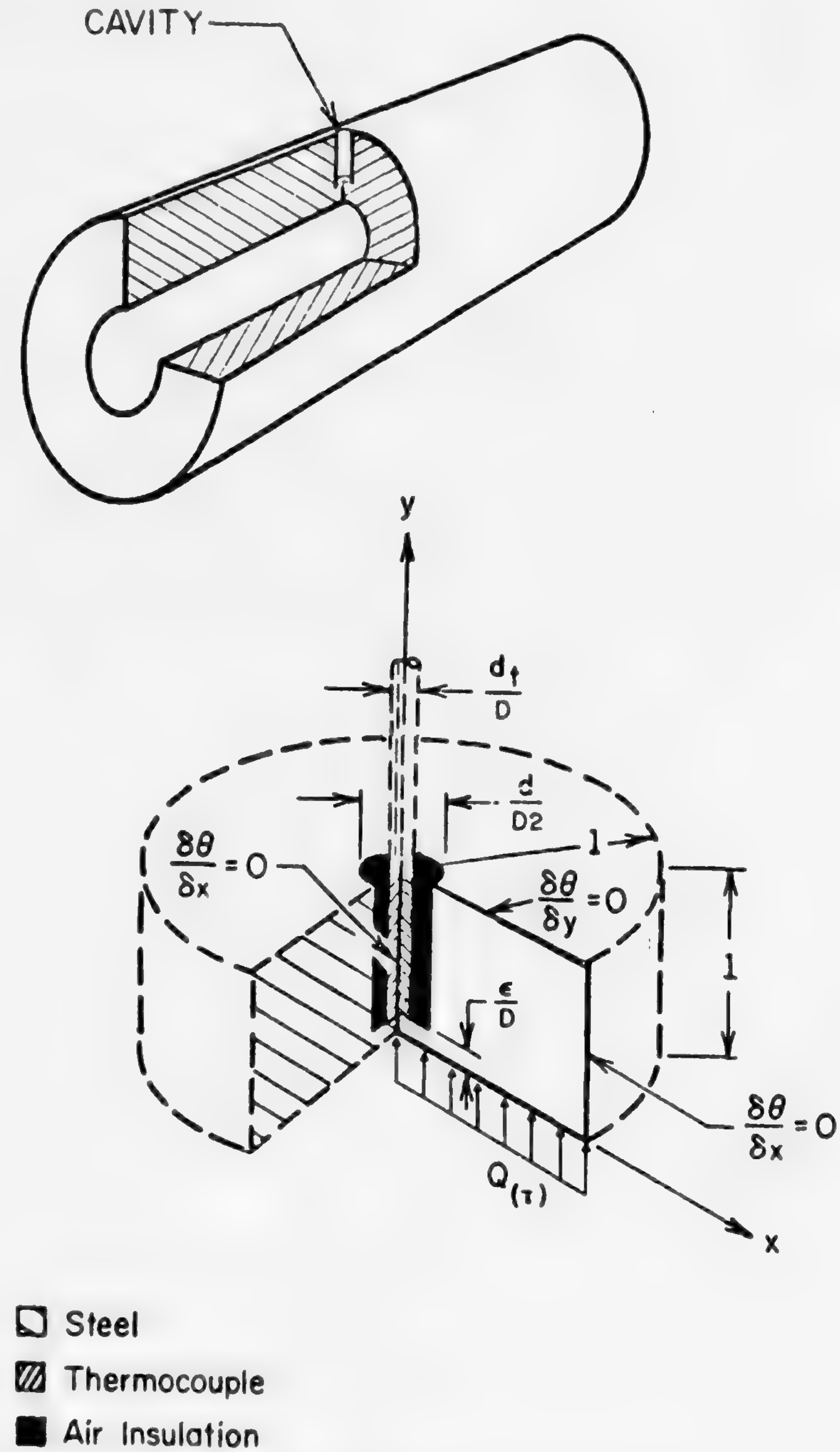

Figure 1 Geometric Representation of Problem

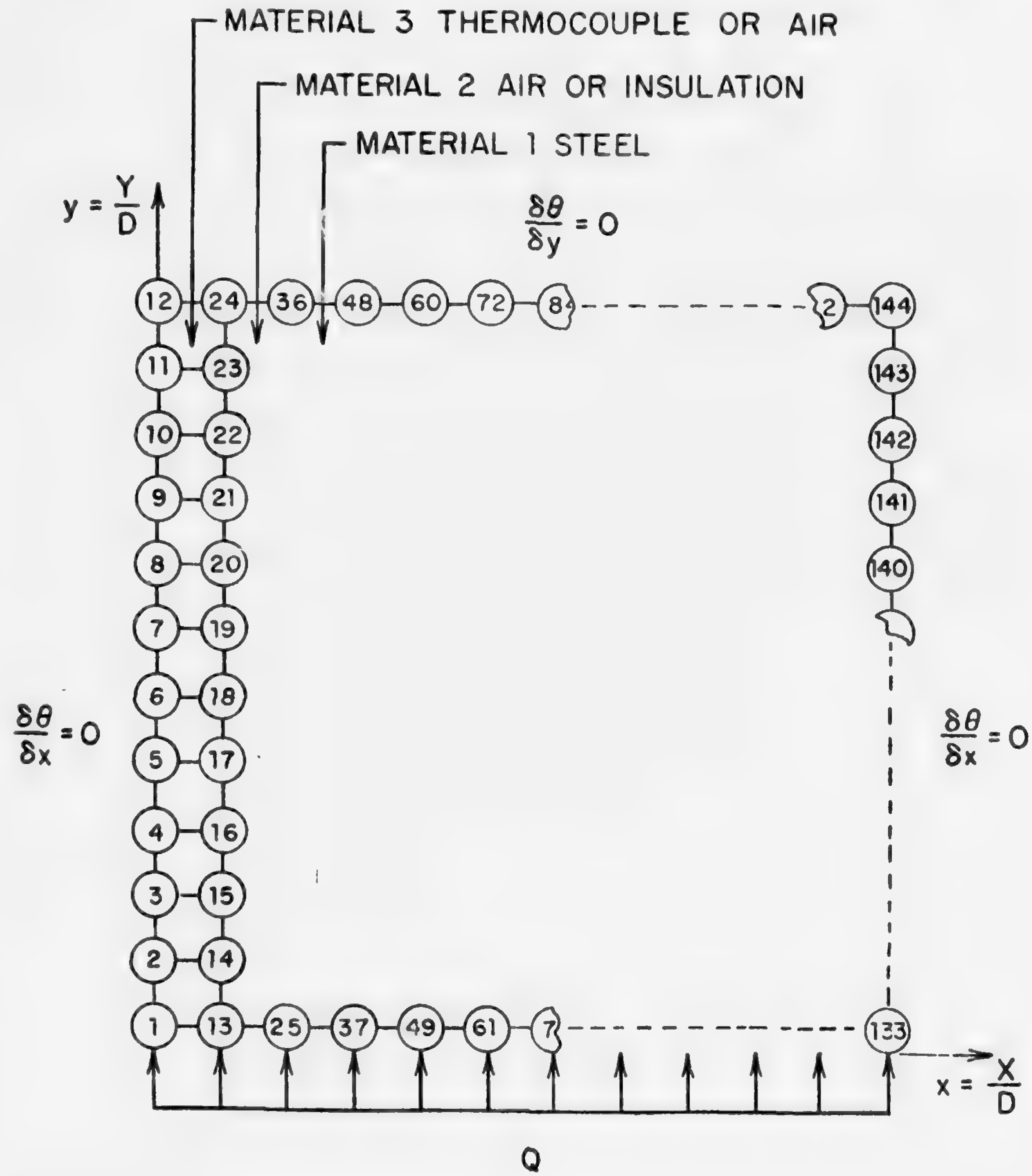

Figure 2 Finite Element Idealization

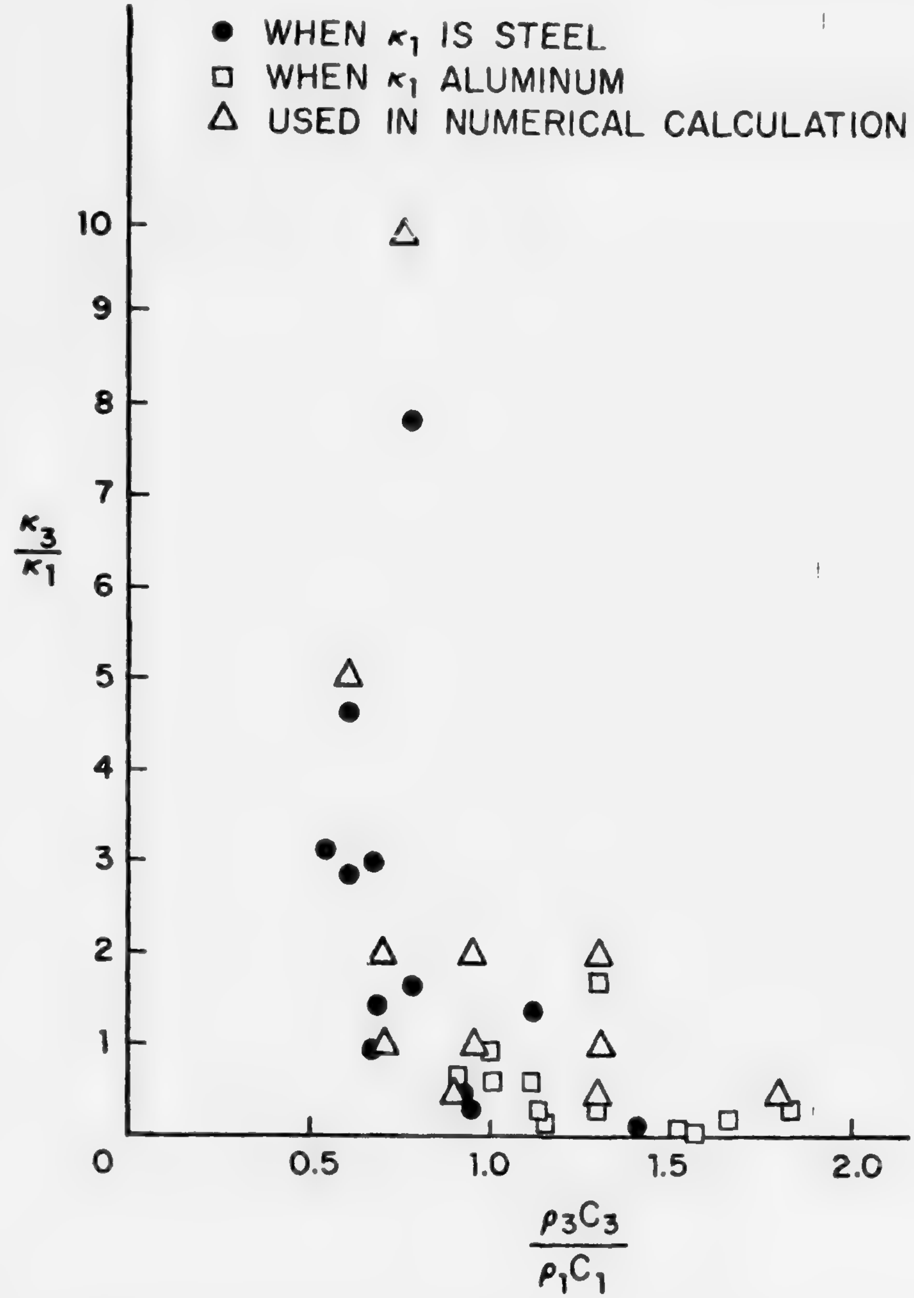

Figure 3b Variation of Thermal Conductivity Ratio and Density - Specific Heat Ratio

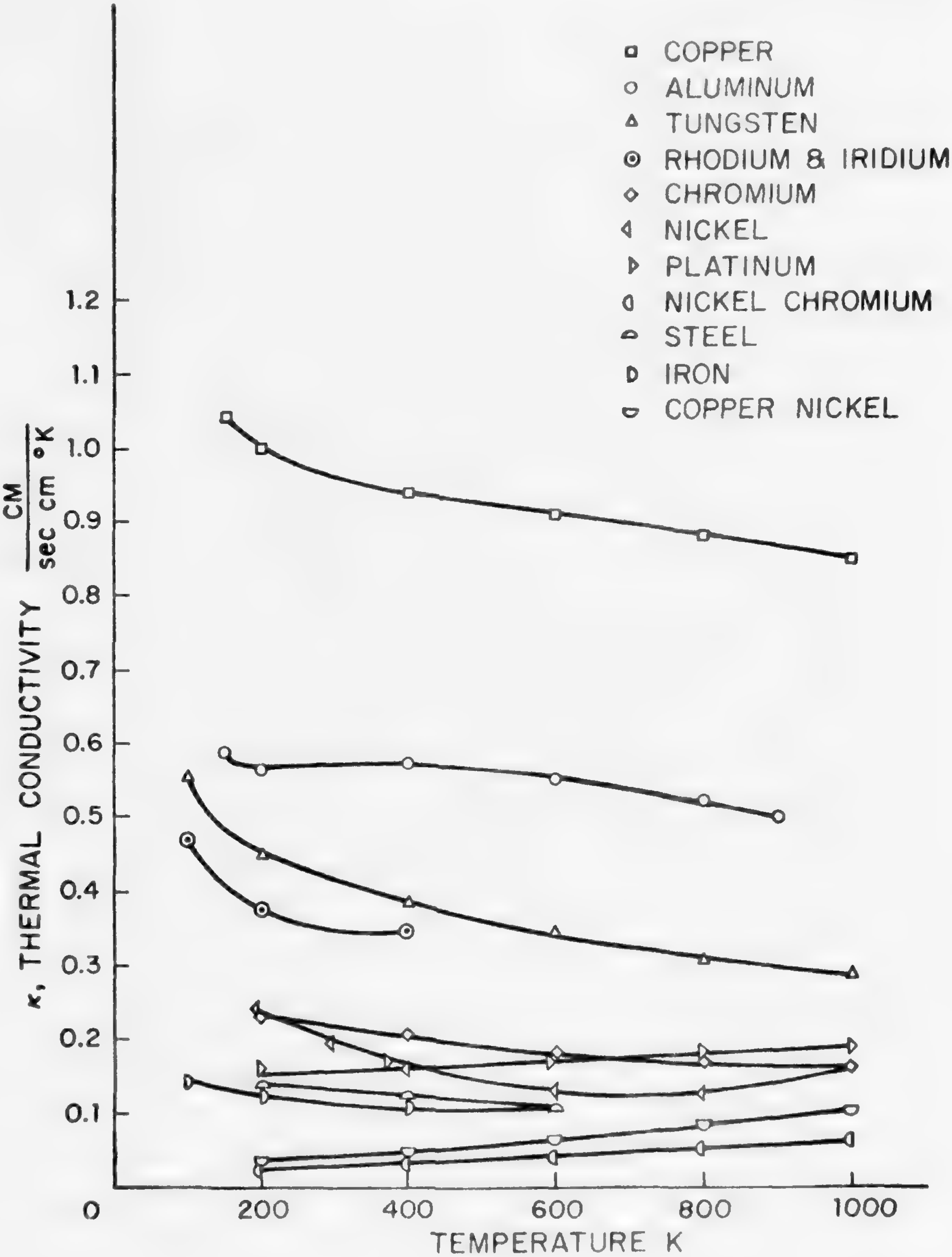

Figure 3a   Thermal Conductivity of Thermocouple Materials

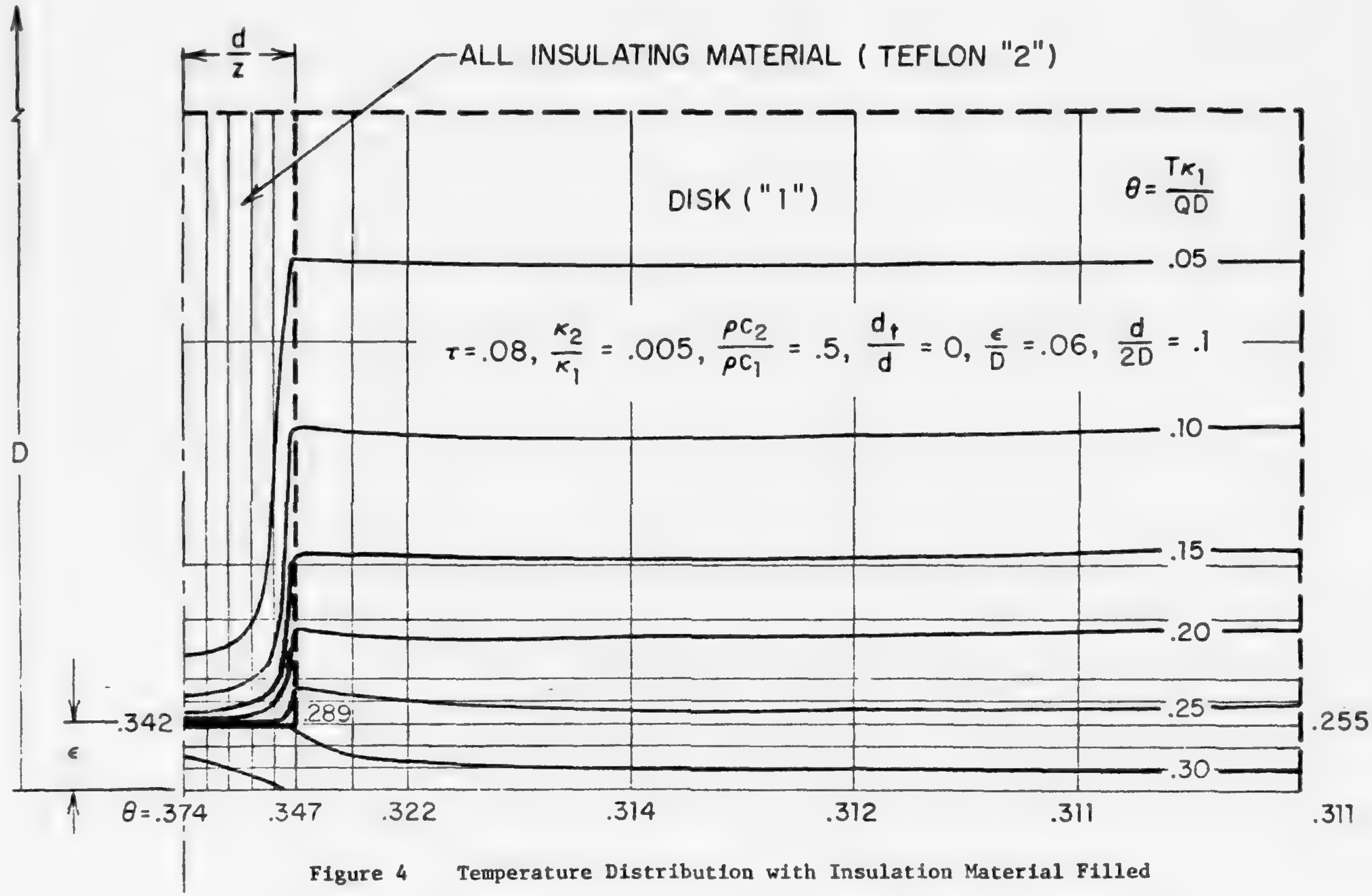

Figure 4 Temperature Distribution with Insulation Material Filled the Cavity

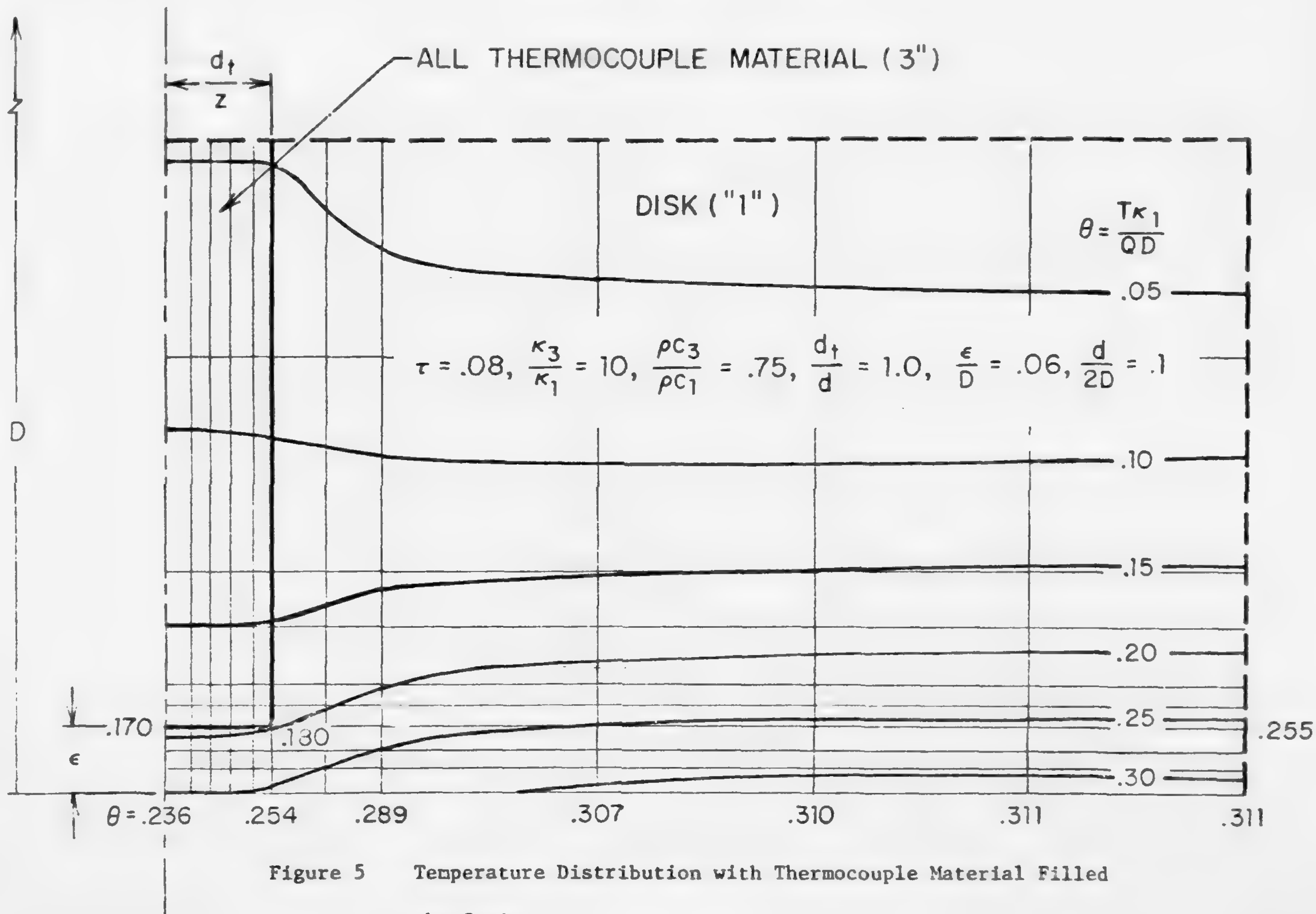

Figure 5 Temperature Distribution with Thermocouple Material Filled the Cavity

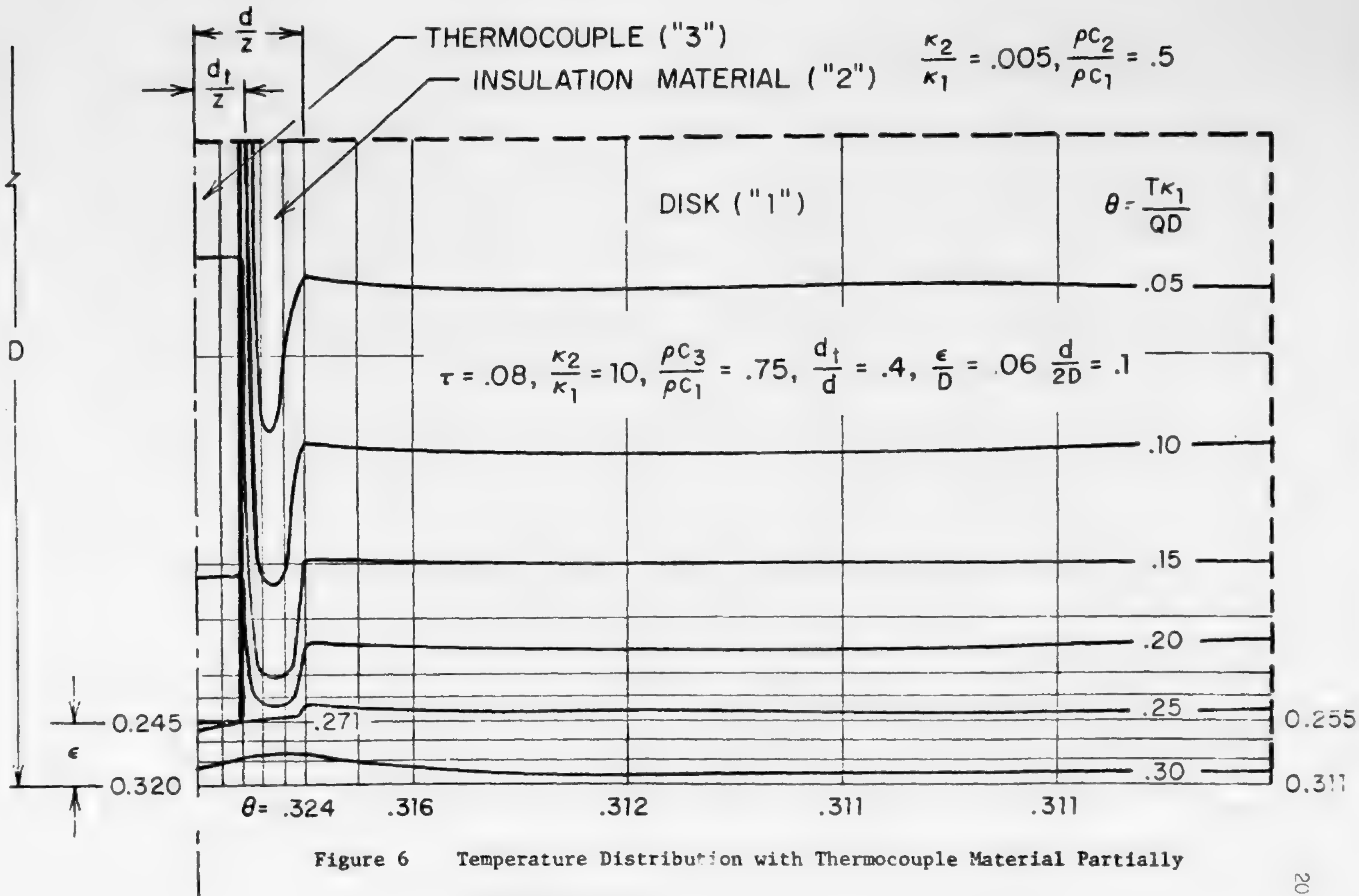

Figure 6 Temperature Distribution with Thermocouple Material Partially Filled the Cavity

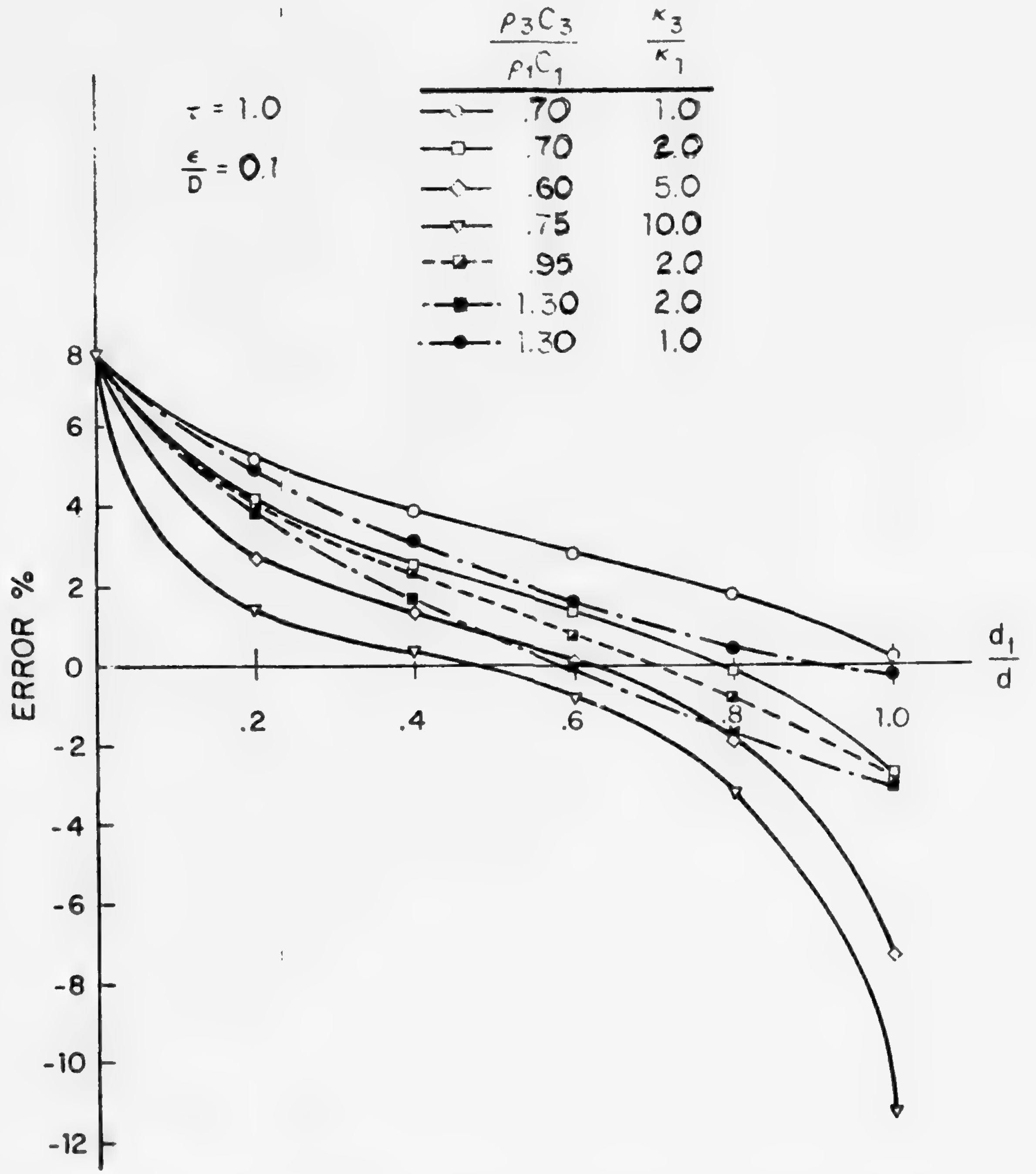

Figure 7 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1, and ρc Ratios $\epsilon/D = 0.1$

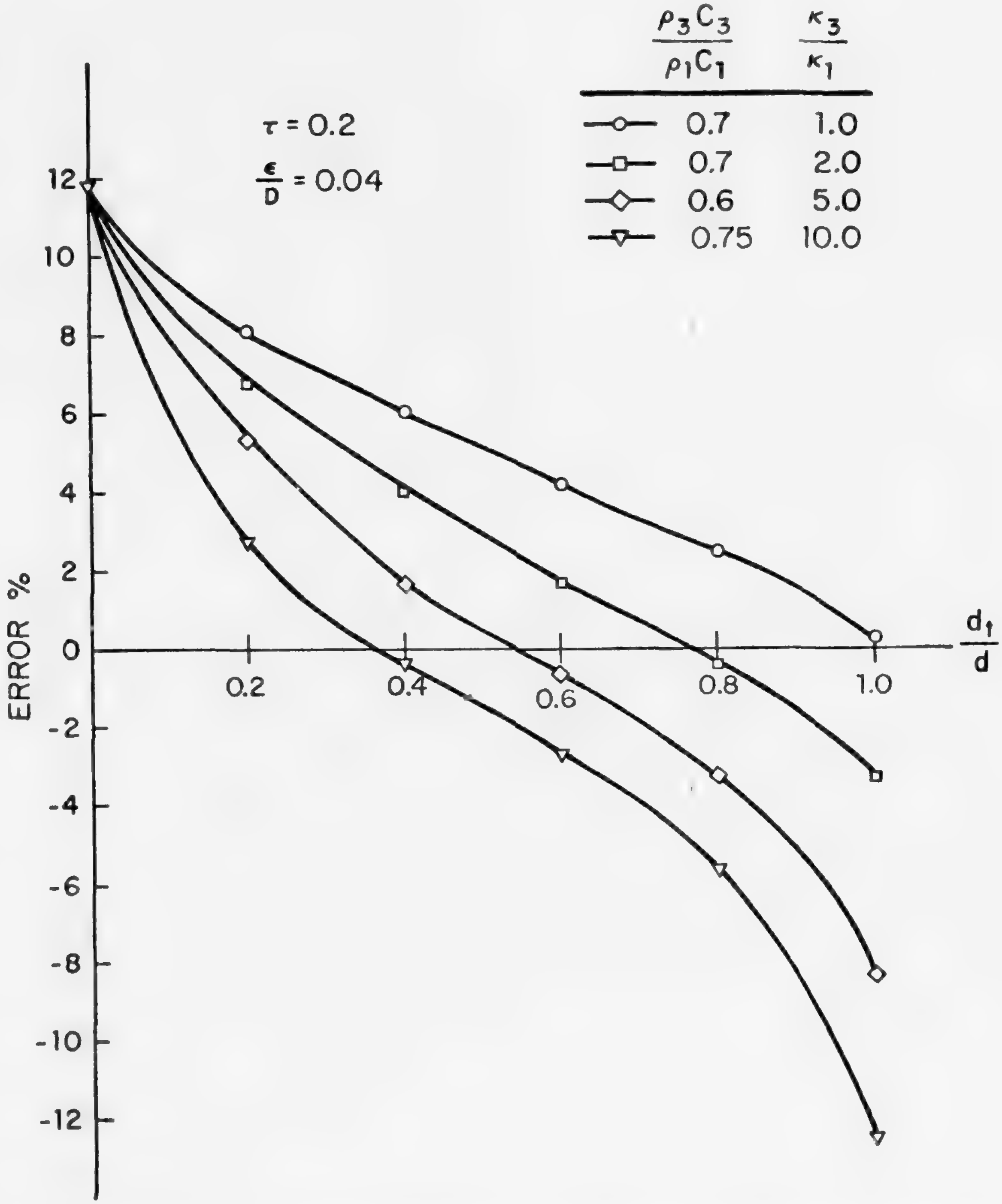

Figure 8 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1 and ρc Ratios $\epsilon/D = 0.04$

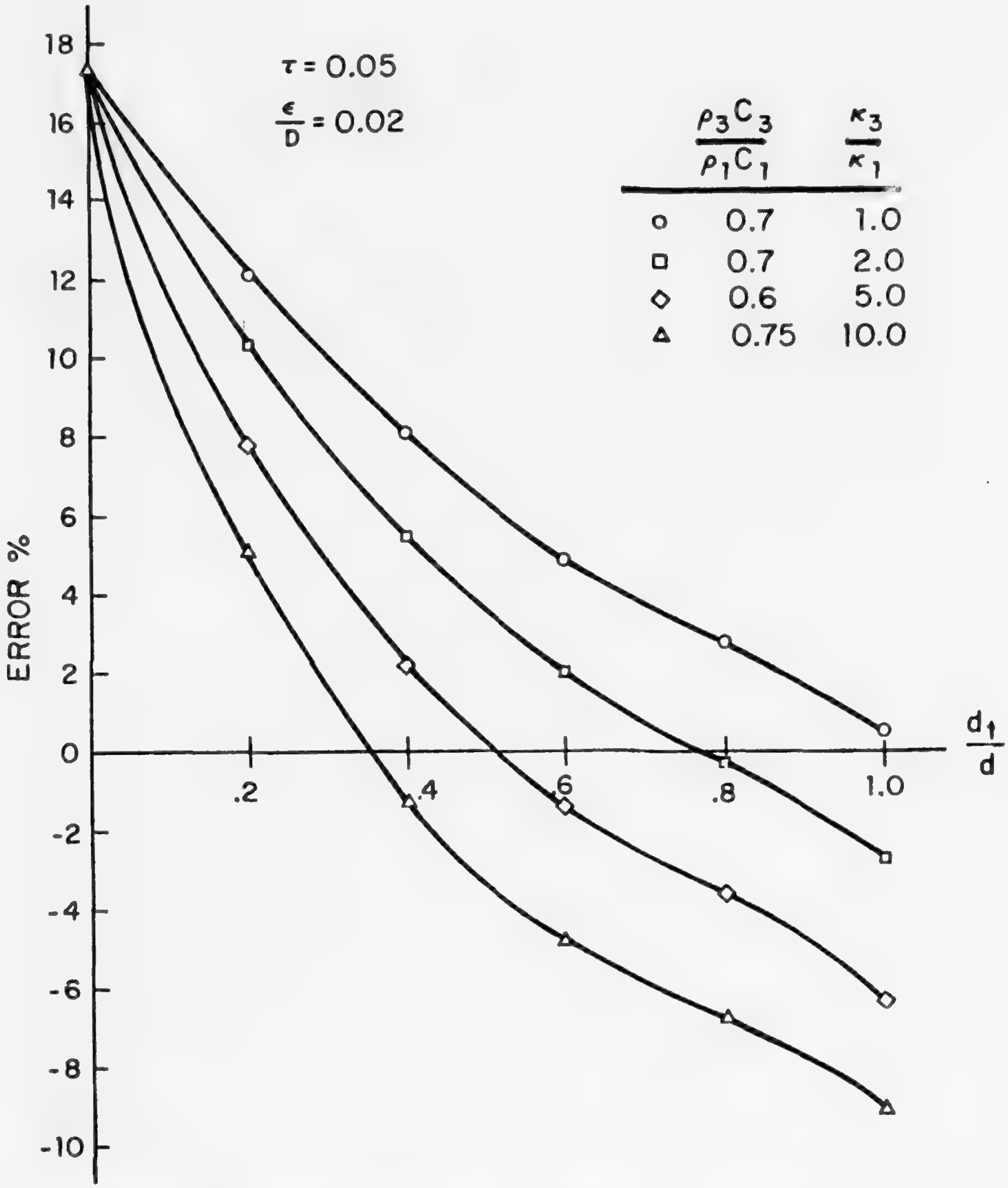

Figure 9 Percentage Error vs. d_t/d Ratio for Various κ_3/κ_1 and ρc Ratios $\epsilon/D = 0.02$

Table 1a

Commonly Used Thermocouples

Types of Thermocouples	Temperature Ranges °F	°C
Copper/Constantan	-300/750	-148/398
Iron/Constantan	-300/1600	-148/871
Chromel/Alumel	-300/2300	-148/1260
Chromel/Constantan	32/1800	0/982
Platinum 10% Rhodium/Platinum	32/2800	0/1537
Platinum 13% Rhodium/Platinum 6% Rh	100/3270	37/1798
Platinel 1813 Platinel 1503	32/2372	0/1300
Iridium/Iridium 60% Rhodium	2552/3326	1400/1830
Tungsten 3% Rhenium/Tungsten 25% Rhenium	50/4000	10/2204
Tungsten/Tungsten 26% Rhenium	60/5072	15/2800
Tungsten 5% Rhenium/Tungsten 26% Rhenium	32/5000	0/2760

Table 1b

Thermal Properties of Thermalcouple Materials

Metal, Insulator (subscript 3)	$K_3 \frac{cal}{sec.\ cm\ K}$	$\frac{\kappa_3}{\kappa\ steel}$	$\frac{\kappa_3}{\kappa\ aluminum}$	$\rho_3 c_3 \frac{cal}{cm^3}$	$\frac{\rho_3 c_3}{\rho c\ steel}$	$\frac{\rho_3 c_3}{\rho c\ aluminum}$
Aluminum	0.554	4.66	1.00	0.65	0.60	1.0
Copper	0.935	7.87	1.69	0.84	0.78	1.30
Chromium	0.201	1.70	0.36	0.84	0.78	1.30
Nickel	0.160	1.35	0.29	1.20	1.12	1.86
Platinum	0.174	1.46	0.31	0.74	0.69	1.14
Steel	0.119	1.0	0.21	1.08	1.00	1.66
Tungsten	0.373	3.14	0.67	0.59	0.55	0.92
Iridium	0.345	2.91	0.62	0.66	0.61	1.01
Rhodium	0.360	3.03	0.65	0.72	0.67	1.12
Rhenium	0.111	0.94	0.20	0.73	0.68	1.14
Nickel-Chromium	0.040	0.34	0.07	1.01	0.94	1.56
Copper-Nickel	0.059	0.50	0.11	0.99	0.92	1.52
Teflon	6×10^{-4}	0.005	0.001	0.54	0.5	0.83
Air	1.26×10^{-4}	0.001	0.0002	0.53	0.49	0.82

Note: The value quoted is the averaged value over 200 to 800 °K whenever data are available.

Table 2

Percentage Error of Temperature at Cavity Base (/D = 0.1)

Table 2.1 — $\kappa_3/\kappa_1 = 0.5$, $\epsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.9	1	1.9	1.4	1.0	0.8	0.6	0.5
	2	3.4	2.5	1.9	1.4	1.1	1.0
	4	5.1	3.8	2.9	2.2	1.7	1.5
	8	6.4	4.7	3.7	2.9	2.3	1.9
	12	6.9	5.1	4.1	3.3	2.6	2.2
	20	7.4	5.5	4.5	3.7	2.9	2.4
	40	7.8	5.9	4.8	4.0	3.2	2.6
	60	7.9	6.0	4.9	4.0	3.2	2.6
	80	7.9	6.0	4.8	3.9	3.2	2.6
	100	7.9	6.0	4.8	3.9	3.2	2.6
1.3	1		1.3	0.9	0.6	0.4	0.3
	2		2.4	1.7	1.1	0.8	0.6
	4		3.7	2.6	1.8	1.2	1.1
	8		4.6	3.4	2.4	1.6	1.5
	12		5.0	3.8	2.7	1.9	1.8
	20		5.4	4.2	3.1	2.2	2.0
	40		5.8	4.5	3.4	2.5	2.3
	60		5.9	4.5	3.4	2.5	2.3
	80		5.9	4.5	3.3	2.5	2.3
	100		5.8	4.5	3.3	2.5	2.3
1.8	1		1.2	0.7	0.4	0.1	0.1
	2		2.3	1.4	0.8	0.4	0.3
	4		3.6	2.3	1.4	0.7	0.6
	8		4.5	3.0	1.9	0.9	1.0
	12		4.9	3.4	2.1	1.1	1.2
	20		5.3	3.8	2.5	1.5	1.6
	40		5.6	4.1	2.8	1.7	1.9
	60		5.7	4.1	2.7	1.7	1.9
	80		5.7	4.1	2.6	1.6	1.9
	100		5.7	4.0	2.6	1.6	1.9

Table 2.2 — $\kappa_3/\kappa_1 = 1.0$, $\epsilon/D = 0.1$

$\rho_3 c_3/\rho_1 c_1$	d_t/d	0.0	0.2	0.4	0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.7	1		1.2	0.8	0.5	0.3	0.2
	2		2.3	1.5	1.0	0.6	0.2
	4		3.4	2.4	1.6	1.0	0.3
	8		4.3	3.1	2.1	1.3	0.3
	12		4.6	3.3	2.3	1.5	0.3
	20		4.9	3.6	2.6	1.6	0.3
	40		5.2	3.8	2.8	1.7	0.2
	60		5.2	3.9	2.8	1.7	0.2
	80		5.2	3.9	2.8	1.8	0.2
	100		5.2	3.9	2.8	1.8	0.2
0.95	1		1.2	0.7	0.4	0.2	0.0
	2		2.2	1.4	0.8	0.3	0.0
	4		3.3	2.2	1.2	0.6	0.0
	8		4.2	2.8	1.7	0.8	0.1
	12		4.5	3.1	1.9	0.9	0.0
	20		4.8	3.4	2.2	1.1	0.0
	40		5.1	3.6	2.3	1.2	0.0
	60		5.1	2.6	2.3	1.2	0.0
	80		5.1	3.6	2.3	1.2	0.0
	100		5.1	3.6	2.3	1.2	0.0
1.3	1		1.1	0.6	0.2	-0.0	-0.1
	2		2.1	1.1	0.5	-0.0	-0.2
	4		3.2	1.8	0.8	0.0	-0.3
	8		4.0	2.4	1.2	0.2	-0.3
	12		4.4	2.7	1.4	0.3	-0.3
	20		4.7	3.0	1.6	0.4	-0.3
	40		5.0	3.2	1.7	0.4	-0.2
	60		5.0	3.2	1.7	0.4	-0.2
	80		5.0	3.2	1.6	0.4	-0.2
	100		5.0	3.1	1.6	0.4	-0.2

Table 3

Percentage Error of Temperature at Cavity (/D = 0.04)

Table 2.3

ρ_3c_3/ρ_1c_1	d_t/d	$\kappa_3/\kappa_1 = 5.0$ 0.0	0.2	0.4	$\epsilon/D = 0.1$ 0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.6	1		0.7	0.1	-0.3	-0.5	-1.0
	2		1.3	0.1	-0.4	-0.9	-1.8
	4		2.1	0.5	-0.6	-1.4	-3.0
	8		2.5	0.6	-0.7	-1.9	-4.5
	12		2.5	0.7	-0.7	-2.0	-5.4
	20		2.6	0.9	-0.4	-2.0	-6.4
	40		2.6	1.2	-0.1	-1.9	-7.2
	60		2.7	1.4	-0.2	-1.8	-7.3
	80		2.7	1.4	-0.2	-1.8	-7.3
	100		2.7	1.4	-0.2	-1.8	-7.3

Table 2.4 $\kappa_3/\kappa_1 = 10$ $\epsilon/D = 0.1$

ρ_3c_3/ρ_1c_1	$\tau \times 10^2$	0.0	0.2	0.4	0.6	0.8	1.0
0.75	1		0.3	-0.4	-0.8	-1.0	-1.4
	2		0.7	-0.6	-1.4	-1.9	-2.7
	4		1.1	-0.8	-2.0	-2.9	-4.6
	8		1.3	-0.9	-2.4	-3.7	-6.8
	12		1.3	-0.8	-2.3	-3.8	-8.3
	20		1.2	-0.4	-1.8	-3.7	-9.9
	40		1.4	0.2	-1.0	-3.3	-11.1
	60		1.4	0.3	-0.9	-3.2	-11.3
	80		1.4	0.4	-0.8	-3.2	-11.3
	100		1.4	0.4	-0.8	-3.2	-11.3

Table 3.1

ρ_3c_3/ρ_1c_1	d_t/d	$\kappa_3/\kappa_1 = 0.5$ 0.0	0.2	0.4	$\epsilon/D = 0.04$ 0.6	0.8	1.0
	$\tau \times 10^2$			Error %			
0.9	.2	1.5	0.9	0.7	0.5	0.4	0.4
	.4	3.2	2.0	1.5	1.1	0.8	0.7
	.8	5.6	3.9	2.8	2.0	1.5	1.3
	1.6	8.2	6.0	4.5	3.3	2.4	1.9
	2.4	9.3	7.0	5.3	3.9	2.9	2.3
	4.0	10.3	7.8	6.1	4.5	3.4	2.7
	8.0	11.2	8.5	6.7	5.2	3.9	3.1
	12.0	11.5	8.8	7.1	5.5	4.2	3.3
	16.0	11.8	9.0	7.3	5.8	4.5	3.4
	20.0	11.9	9.1	7.5	6.0	4.6	3.5
1.3	.2		0.8	0.5	0.3	0.2	0.2
	.4		1.9	1.2	0.8	0.5	0.4
	.8		3.7	2.5	1.6	1.0	0.8
	1.6		5.8	4.0	2.6	1.6	1.3
	2.4		6.8	4.9	3.2	2.1	1.6
	4.0		7.6	5.5	3.8	2.5	2.0
	8.0		8.4	6.3	4.4	2.9	2.5
	12.0		8.6	6.6	4.7	3.2	2.7
	16.0		8.8	6.9	5.0	3.4	2.9
	20.0		9.0	7.0	5.2	3.6	3.0
1.8	.2		0.7	0.4	0.2	0.1	0.0
	.4		1.7	0.9	0.5	0.2	0.1
	.8		3.4	2.1	1.1	0.5	0.3
	1.6		5.5	3.5	2.0	0.9	0.6
	2.4		6.5	4.3	2.5	1.2	0.9
	4.0		7.4	5.1	3.1	1.6	1.2
	8.0		8.2	5.8	3.6	1.9	1.7
	12.0		8.5	6.1	3.9	2.1	2.0
	16.0		8.7	6.3	4.1	2.3	2.2
	20.0		8.8	6.5	4.3	2.5	2.4

Table 3.2 $\kappa_3/\kappa_1 = 1.0$ $\epsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d / $\tau \times 10^2$	0.0	0.2	0.4	0.6	0.8	1.0
		Error %					
0.7	0.2		0.8	0.5	0.3	0.2	0.1
	0.4		1.8	1.1	0.7	0.4	0.3
	0.8		3.4	2.2	1.4	0.8	0.4
	1.6		5.4	3.6	2.2	1.3	0.4
	2.4		6.3	4.3	2.7	1.6	0.4
	4.0		7.1	5.0	3.2	1.9	0.4
	8.0		7.6	5.5	3.7	2.2	0.4
	12.0		7.9	5.8	4.0	2.4	0.4
	16.0		8.0	5.9	4.1	2.4	0.4
	20.0		8.1	6.1	4.2	2.5	0.3
0.95	0.2		0.7	0.4	0.2	0.1	0.0
	0.4		1.6	0.9	0.5	0.2	0.0
	0.8		3.3	1.9	1.0	0.4	0.1
	1.6		5.2	3.3	1.8	0.7	0.1
	2.4		6.1	3.9	2.2	0.9	0.1
	4.0		6.9	4.6	2.6	1.1	0.1
	8.0		7.5	5.1	3.1	1.4	0.1
	12.0		7.7	5.4	3.3	1.6	0.1
	16.0		7.9	5.6	3.5	1.6	0.1
	20.0		8.0	5.7	3.6	1.7	0.1
1.3	0.2		0.6	0.3	0.0	-0.1	-0.1
	0.4		1.5	0.7	0.2	-0.1	-0.2
	0.8		3.1	1.6	0.6	-0.1	-0.3
	1.6		5.0	2.8	1.2	0.1	-0.4
	2.4		5.9	3.5	1.5	0.2	-0.4
	4.0		6.7	4.1	1.0	0.3	-0.4
	8.0		7.3	4.6	2.2	0.4	-0.4
	12.0		7.6	4.9	2.5	0.5	-0.4
	16.0		7.7	5.1	2.7	0.6	-0.4
	20.0		7.9	5.2	2.8	0.7	-0.3

Table 3.4

Table 3.3 $\kappa_3/\kappa_1 = 5$ $\epsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	d_t/d / $\tau \times 10^2$	0.0	0.2	0.4	0.6	0.8	.10
		Error %					
0.6	0.2		0.3	-0.1	-0.3	-0.4	-0.6
	0.4		0.9	0.0	-0.5	-0.8	-1.1
	0.8		2.0	0.4	-0.6	-1.2	-2.1
	1.6		3.4	0.9	-0.7	-1.8	-3.2
	2.4		4.1	1.2	-0.8	-2.1	-4.0
	4.0		4.6	1.4	-0.9	-2.5	-5.0
	8.0		4.7	1.4	-1.1	-3.0	-6.4
	12.0		4.7	1.4	-1.0	-3.2	-7.3
	16.0		4.7	1.6	-0.8	-3.2	-7.9
	20.0		4.7	1.7	-0.6	-3.2	-8.3

Table 3.4 $\kappa_3/\kappa_1 = 10$ $\epsilon/D = 0.04$

$\rho_3 c_3/\rho_1 c_1$	$\tau \times 10^2$	0.0	0.2	0.4	0.6	0.8	.10
0.75	0.2		0.0	-0.4	-0.6	-0.7	-0.9
	0.4		0.3	-0.6	-1.1	-1.4	-1.7
	0.8		1.0	-0.7	-1.7	-2.3	-3.0
	1.6		2.1	-0.8	-2.5	-3.5	-4.6
	2.4		2.6	-0.8	-2.9	-4.2	-5.7
	4.0		2.9	-0.9	-3.4	-5.0	-7.2
	8.0		2.9	-1.0	-3.7	-5.8	-9.5
	12.0		2.8	-0.8	-3.4	-5.8	-11.0
	16.0		2.8	-0.5	-3.1	-5.7	-11.9
	20.0		2.8	-0.3	-2.7	-5.6	-12.5

Table 4

Percentage Error of Temperature at Cavity Base (ϵ/D = 0.02)

Table 4.1 κ_3/κ_1 = 0.5 ϵ/D = 0.02

ρ_3c_3/ρ_1c_1	d_t/d / $t \times 10^2$	0.0	0.2	0.4	0.6	0.8	1.0
			Error %				
1.3	0.05		0.1	0.1	0.1	0.1	0.0
	0.10		0.8	0.4	0.2	0.2	0.1
	0.20		2.0	1.2	0.7	0.5	0.4
	0.40		4.5	2.7	1.6	1.0	0.8
	0.60		6.5	4.0	2.3	3.4	1.1
	1.00		8.9	5.6	3.3	2.0	1.5
	2.00		11.4	7.6	4.7	2.8	2.2
	3.00		12.4	8.5	5.4	3.3	2.6
	4.00		12.9	9.0	5.8	3.6	2.8
	5.00		13.1	9.3	6.0	3.7	3.0

Table 4.2 κ_3/κ_1 = 1 ϵ/D = 0.02

ρ_3c_3/ρ_1c_1	d_t/d / $t \times 10^2$	0.0	0.2	0.4	0.6	0.8	1.0
0.7	0.05		0.3	0.2	0.1	0.1	0.1
	0.10		0.8	0.5	0.3	0.2	0.2
	0.20		1.9	1.1	0.7	0.4	0.3
	0.40		4.3	2.5	1.5	0.8	0.5
	0.60		6.1	3.6	2.1	1.1	0.5
	1.00		8.3	5.1	2.9	1.5	0.6
	2.00		10.6	6.8	3.9	2.1	0.6
	3.00		11.4	7.5	4.4	2.4	0.5
	4.00		11.8	7.8	4.7	2.5	0.5
	5.00		12.1	8.1	4.9	2.8	0.5
0.95	0.05		0.2	0.1	0.0	0.0	0.0
	0.10		0.7	0.3	0.1	0.1	0.0
	0.20		1.8	0.9	0.4	0.1	0.0
	0.40		4.0	2.1	1.0	0.3	0.1
	0.60		5.8	3.1	1.5	0.5	0.1
	1.00		8.0	4.5	2.2	0.8	0.1
	2.00		10.3	6.1	3.1	1.2	0.1
	3.00		11.2	6.8	3.5	1.4	0.1
	4.00		11.6	7.2	3.8	1.5	0.1
	5.00		11.8	7.4	4.0	1.7	0.1
1.3	0.05		0.2	0.0	-0.0	-0.1	-0.1
	0.10		0.5	0.1	-0.1	-0.1	-0.1
	0.20		1.6	0.6	0.1	-0.2	-0.3
	0.40		3.7	1.6	0.5	-0.2	-0.4
	0.60		5.4	2.5	0.8	-0.1	-0.5
	1.00		7.6	3.8	1.4	-0.0	-0.5
	2.00		10.0	5.4	2.1	0.2	-0.5
	3.00		10.9	6.0	2.5	0.3	-0.5
	4.00		11.3	6.4	2.7	0.4	-0.5
	5.00		11.5	6.7	2.9	0.4	-0.5

Table 4.3 κ_3/κ_1 = 10 ϵ/D = 0.02

ρ_3c_3/ρ_1c_1	d_t/d / $t \times 10^2$	0	0.2	0.4	0.6	0.8	1.0
0.75	0.05	0.7	0.0	-0.26	-0.33	-0.37	-0.41
	0.10	1.7	0.0	-0.48	-0.66	-0.76	-0.86
	0.20	4.9	0.2	-0.72	-0.93	-1.40	-1.61
	0.40	9.0	1.2	-0.85	-1.78	-2.28	-2.71
	0.60	11.6	2.0	-0.86	-2.20	-2.89	2.50
	1.00	13.2	3.2	-0.84	-2.78	-3.76	-4.66
	2.00	15.8	4.5	-0.88	-3.66	-5.06	-6.44
	3.00	16.7	4.9	-0.99	-4.20	-5.84	-7.56
	4.00	17.1	5.1	-1.10	-4.57	-6.38	-8.42
	5.00	17.4	5.1	-1.20	-4.80	-6.75	-9.12

PART II IMPROVED ACCURACY IN THE PREDICTION OF SURFACE HEAT FLUX AND TEMPERATURE BY AN INTRINSIC THERMOCOUPLE

1. INTRODUCTION

In the study of transient heat transfer, many experimental difficulties may arise if heat flux sensors or thermocouples are installed direct at the surface of a body. For example, a probe may be damaged by a piston or a projectile sliding over a cylinder or barrel. A probe on a melting and ablative surface of heat shield can be easily destroyed because of high temperature. Furthermore a surface probe exposed to both radiative and convective environment may measure an erroneous surface heat flux and temperature if the probe has a different radiative property from that of the measured surface. In these circumstances, calculation of the transient surface heat flux and the surface temperature can be achieved by inverting a temperature history measured at some location inside the body.

In general, the prediction of a surface heat flux and temperature by the measured data at some location interior to a body is known as the "inverse problem". Many configurations, such as spheres, cylinders, and slabs, had been studied by many workers and many methods such as numerical, graphical, series, convolution integral, and Laplace transforms were used. Stolz [1], Beck [2] and Williams and Curry [3], considered the numerical inversion of the integral solution for semi-infinite and other bodies. In this method, care is required in selecting a time interval in order to achieve a stable solution. Carslaw and Jaeger [4], Burggraf [5], Koveryanov [6], and Shumakov [7], respectively considered different series approaches in which generally the local heat flux at an interior location and their higher derivaties are required. However, it is difficult to measure experimentally or to process the measured data for the derivative

of the temperature. Sparrow, Haji-Sheikh, and Lundgren [8], Imber and Kahn [9], Imber [10], Sabherwal [11], Masket and Vastano [12], Deverall and Channapragada [13] and Chen and Thomsen [14] applied the transform method. In these works, the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after an expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [14] introduced a polynomial in terms of an error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. In thier study, the cylindrical thickness was assumed to be relatively thick such that the temperature at a large distance from the heating surface remains constant. Therefore, only one interior temperature response near the surface was needed in the experimental measurement. Their inversion solution however was valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. Chen and Chiou [15] studied the inversion problem for the case of a semi-infinite slab or a thick slab using a Laplace transformation. The exact solution was obtained from the inverse Laplace transform for any time interval. It was then shown that their analysis may be approximately applied to the case of the hollow cylinder if the interior temperature response is measured at a location close to the inner wall.

This report presents (a) the improved numerical solution of the inversion solution reported by Chen and Chiou [15] and (b) a further demonstration of the capability of the solution.

The theoretical analysis of Chen and Chiou [15] is recapitulated in Appendix I in which the surface heat flux and temperature is predicted by inverting a temperature history measured at some location inside the solid body. The inversion solution is obtained by invoking Laplace transformation. Both the surface heat flux and temperature are given by Eqs. (19) and (20) in Appendix I.

It was thought that the accuracy of the computer program generated for the solution in the previous report by Chen and Chiou [15] can be improved further for the following reasons. First, the coefficients b_n (see Appendix Eq. (11)) in the previous formulation has a dimension of temperature. Therefore the determination of the coefficients depends on the temperature range of each particular experiment. It was found that the absolute value of the coefficients b_n in some cases can become as large as an order of 10^{44}. Therefore during the subsequently numerical manipulation in the computer program error due to round off and the standard fixed up when an underflow occurred may become appreciable. To remedy this difficulty the dimensionless formulation is introduced in the analysis (Appendix I) in which the coefficient b_n is also made dimensionless. As a result the magnitude of the coefficient b_n can be greatly reduced. Secondly, the double precision format was not used throughout the previous computer program. It is felt that further accurate results may be obtained if the double precision format is adopted in the program.

In Section II the new computer solution is shown to be indeed more accurate. Also in Section III the solution is shown to be capable of predicting a case involving a periodic surface heat flux or periodic temperature variation.

II. RESULTS OF THE IMPROVED COMPUTER PROGRAM

The previous computer program of Chen and Chiou [15] was recasted in dimensionless form and written in the double precision format. The new computer program is listed in Appendix II. The results predicted by the new and previous computer program are given in Appendix III and shown in Figure 1 for the case of the constant surface heat flux. This is the case in which a steel slab initially at a uniform temperature is suddenly subjected to a constant heat flux Q at one of the surfaces and kept at the initial temperature on the other surface. Figure 1 shows the solution predicted by inverting the temperature response at an interior of the slab from the new and previous computer program. This solution predicted by the new and previous programs used the ten term representation for the thermocouple response. The comparison clearly shows the improvement of the new solution over the previous one. Except for the short time duration the solution with the new program reduces the error to only one half of the error of the previous program i.e., an error of less than one percent. In the short time period the solution exhibits a Gibbs phenomenon* because of the discontinuity of the surface temperature gradient occurred at initial condition. The solution shows a 17% of initial overshot of heat flux and then a 7.8% of undershot before the solution approaches the constant heat flux. It should be remarked that Gibbs phenomenan is artificially

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* Gibbs phenomena [3]: for a sequence of transformation Tn(t), n = 1, 2,... of a function q(t) (here q(t) = constant) if the interval $[\lim_{t\to t_o} \inf_{n\to\infty} Tn(t), \lim_{t\to t_o} \sup_{n\to\infty} Tn(t)]$ contains points outside the interval $[\lim_{t\to t_o} \inf q(t), \lim_{t\to t_o} \sup q(t)]$ then the sequence is said to exhibit a Gibbs phenomena.

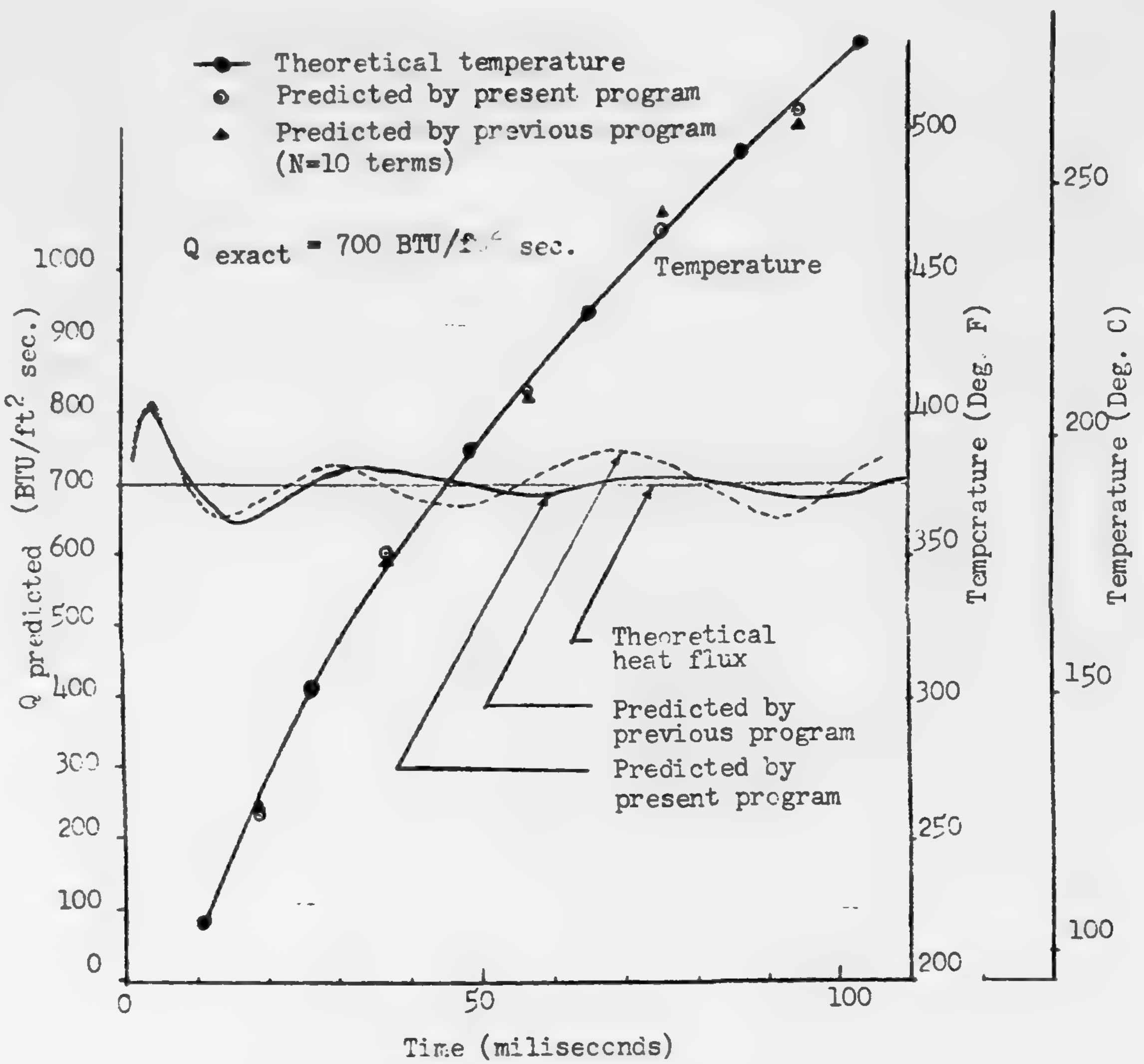


Figure 1 Comparison of Previous and present Programs

introduced due to the idealization of the initial condition. In most practical situations the surface heat flux will be continuous. Therefore, Gibbs phenomenon will not appear.

Figure 2 (see Table 1 also) shows the comparison between the solutions for the constant heat flux case with 10 and 20 term representation for the thermocouple response. One sees that the solution with 20 term representation after the initial Gibbs phenomenon quickly approaches the expected constant heat flux solution with a negligible error of less than 0.14 percent. This shows the accuracy of the new computer program. From Figure 2 one also observes that both overshoot and undershoot of Gibbs phenomenon are smaller for the 20 term representation. Additionally the points of the overshoot and the undershoot has moved to near the zero time which agrees with the characteristic of Gibbs phenomenon. According to Gibbs phenomenon the point of overshoot shoot should approach the initial zero if the number of term of the series which represent the thermocouple response is increased to infinite.

VII. VERIFICATION OF OSCILLATORY SOLUTION

As a measure of applicability of the present inversion solution, a test problem was solved for the case of a slab subjected to a preiodic surface temperature variation on one surface and held to the initial temperature on the other. The analytic solution for the problem is given in Appendix IV where a more suitable form of the solution than the one given by Carslaw and Jaeger [4] is derived and tabulated for the thermocouple response at one tenth of the slab thickness from the surface. The

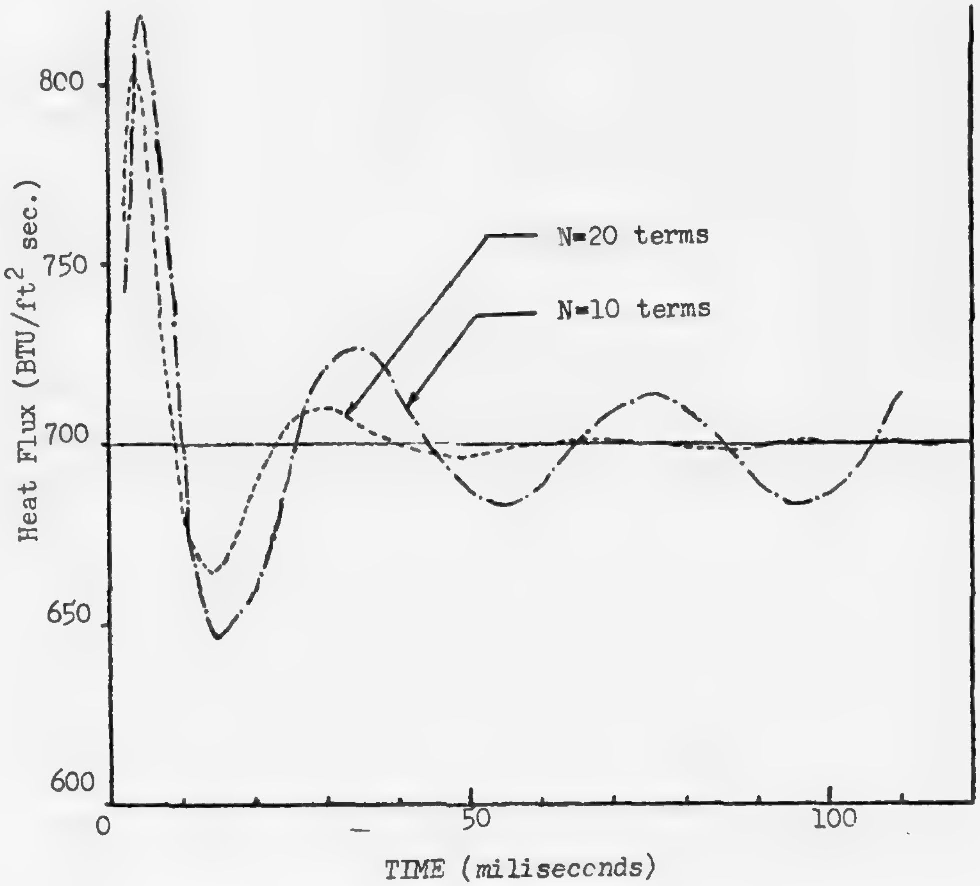


Figure 2 Comparison of Different Polynomial Representations

TABLE 1

Comparision of Inversion Prediction and Exact Solution

N	t	f(t) $\theta(1,t)$	$\theta(0,t)$ EXACT	$\theta(0,t)$ PREDICTED	ERROR* %	$\frac{\partial\theta}{\partial x}(0,t)$ EXACT	$\frac{\partial\theta}{\partial x}(0,t)$ PREDICTED	ERROR* %
20	0.1	0.0624	0.3758	0.3773	+0.399	1.0000	0.9873	-1.27
	0.2	0.1628	0.5315	0.5314	+0.019	1.0000	1.0014	+0.14
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0001	+0.01
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	1.0000	0.00
	0.8	0.6183	1.0629	1.0628	-0.009	1.0000	0.9996	-0.04
	1.0	0.7353	1.1884	1.1878	-0.05	1.0000	1.9986	-0.14
10	0.1	0.0624	0.3758	0.4048	+7.71	1.0000	1.0796	+7.96
	0.2	0.1628	0.5315	0.5284	-0.58	1.0000	0.9391	-6.09
	0.4	0.3395	0.7516	0.7516	0.00	1.0000	1.0179	+1.79
	0.6	0.4882	0.9205	0.9205	0.00	1.0000	0.9923	-0.77
	0.8	0.6183	1.0629	1.0626	-0.028	1.0000	1.0043	+0.43
	1.0	0.7353	1.1884	1.1887	+0.025	1.0000	0.9963	-0.37

*ERROR % = ((PREDICTED)-(EXACT))/(EXACT)

surface is subjected to a periodic temperature variation with a period of 8 milliseceonds. Fifteen data points of the temperature response at the thermocouple location are then input to the inversion program for prediction of the surface temperature and heat flux. The result is shown in Figure 3 and Appendix III where the data symbol "2" denotes the thermocouple response and "1" the surface temperature. The accuracy of the inversion program is shown in Table 2. Except for the extremely short time period of 0.4 milliseconds the prediction by the inversion program with 15 term representation is within 2 percent of error.

The accuracy can be improved more if more data points are used. Figure 4 shows the predicted surface heat flux which we were unable to compute from the series solution (Eq. (5) of Appendix IV). This demonstrates the versatility of the inversion solution.

IV. APPLICATION OF THE INVERSION PROGRAM

Three sets of data (see Appendix III) provided by Rock Island Arsenal for the temperature response of a thermocouple embedded in a M60 gun barrel were utilized to evaluate the inversion solution. The inversion prediction for the surface heat flux from all three sets of data were extremely high when compared with other known data calculated by Chen and Chiou [15]. Since the program correctly predicted the surface heat flux for other sets of experimental data it was judged that the three sets of data may contain inaccurate initial time. For most experimentations the recording instrument is likely to experience some delay in responding to the extremely fast transient heat flux typical in gun bores. Therefore an advanced shift of time of 2 milliseconds in the data was tested. The

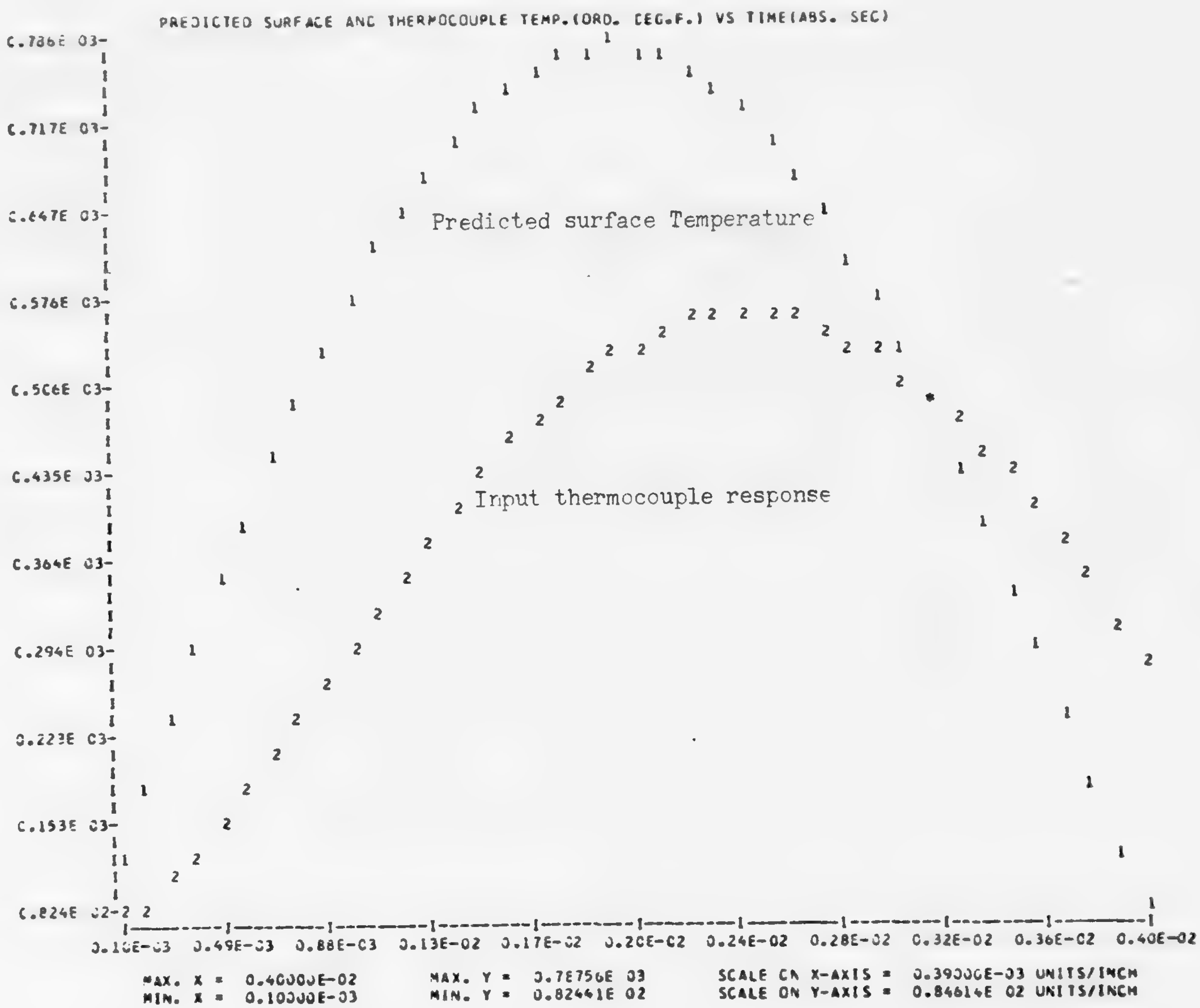


Figure 3 Predicted Surface Temperature

Table 2

Comparison of Inversion Prediction and Exact Solution

Time (sec.)	Surface Temperature (theoretical)	Surface Temperature (predicted)	Error* %
0.0002	189.5041	193.4060	+3.563
0.0004	296.3119	301.4311	+2.367
0.0006	397.7934	403.8998	+1.921
0.0008	491.4497	498.3988	+1.689
0.0010	574.9747	582.5683	+1.534
0.0012	646.3119	654.3358	+1.417
0.0014	703.7046	711.9396	+1.321
0.0016	745.7396	753.9650	+1.236
0.0018	771.3818	779.3802	+1.157
0.0020	780.0000	787.5615	+1.08
0.0022	771.3818	778.3093	+1.00
0.0024	745.7396	751.8527	+0.918
0.0026	703.7046	408.8443	+0.874
0.0028	646.3119	650.3441	+0.712
0.0030	574.9747	577.7933	+0.569
0.0032	491.6697	492.9790	+0.372
0.0034	397.7934	397.9893	+0.0616
0.0036	296.3119	295.2761	-0.679
0.0038	189.5041	188.8522	-0.595

*Error = ((Predicted - (Exact))/(Exact)) x 100

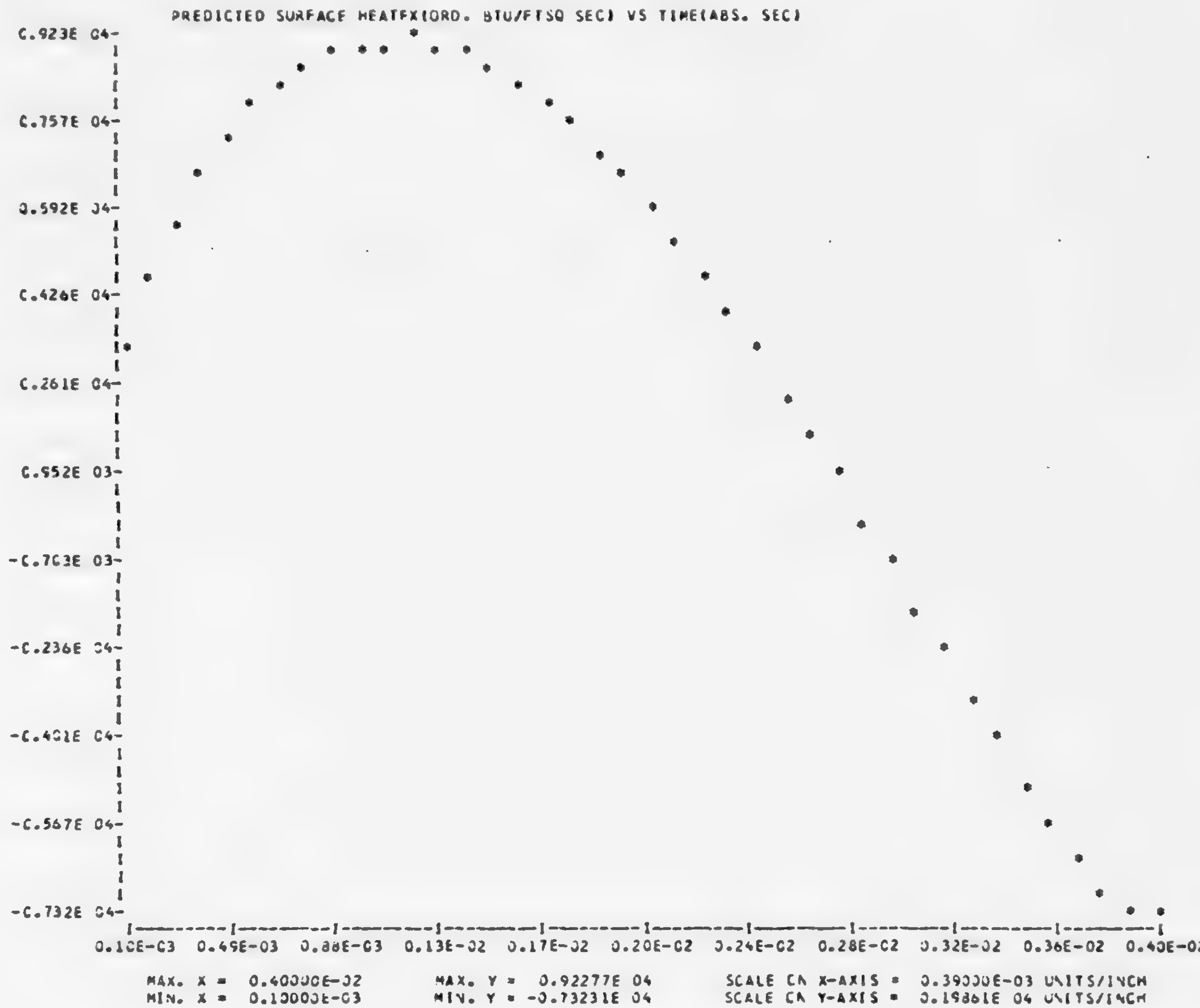
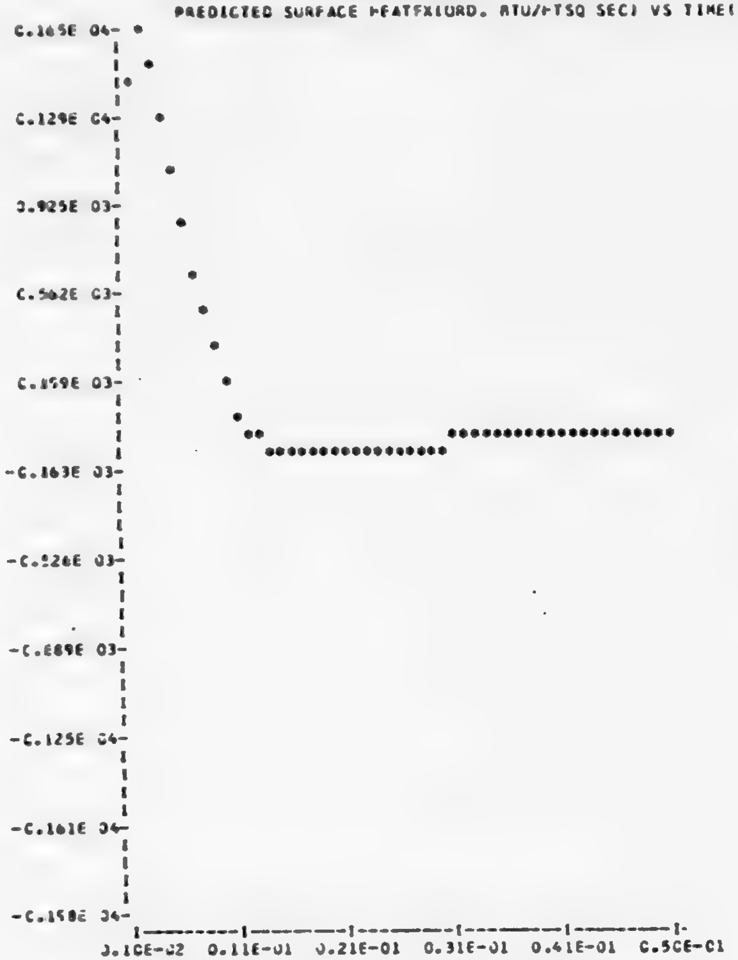


Figure 4 Predicted Surface Heat Flux

result is shown in Appendix III-3. Figures 5, 6 and 7 respectively show the predicted maximum heat flux of 1650, 1235 and 2695 Btu/ft^2 sec. approximately at 2 milliseconds after the firing, while Figures 8, 9 and 10 show that the surface temperature are 273, 234 and 396 degree F. approximately at 6 milliseconds after the firing.

Figure 5 M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH

PREDICTED SURFACE HEATFX(URD. BTU/FTSQ SEC) VS TIME(ABS. SEC)

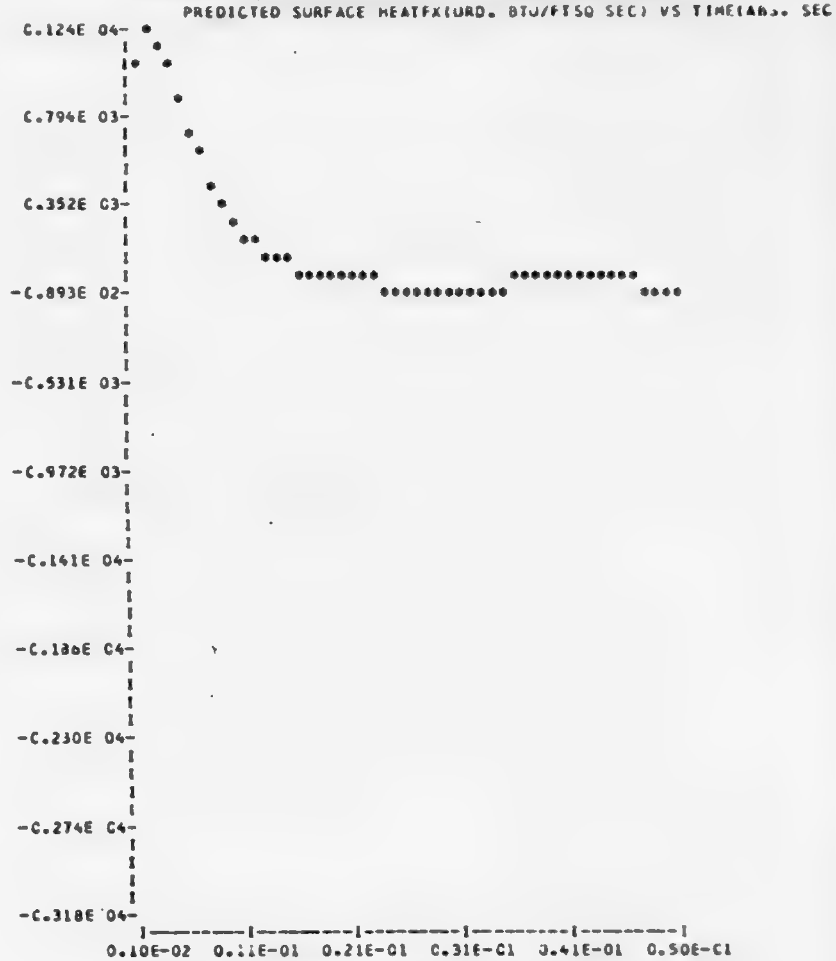


MAX. X = 0.10000E 00 MAX. Y = 0.16504E 04
MIN. X = 0.10000E-02 MIN. Y = -0.19771E 04

SCALE CN X-AXIS = 0.99000E-02 UNITS/INCH
SCALE CN Y-AXIS = 0.43530E 03 UNITS/INCH

Figure 6 M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH

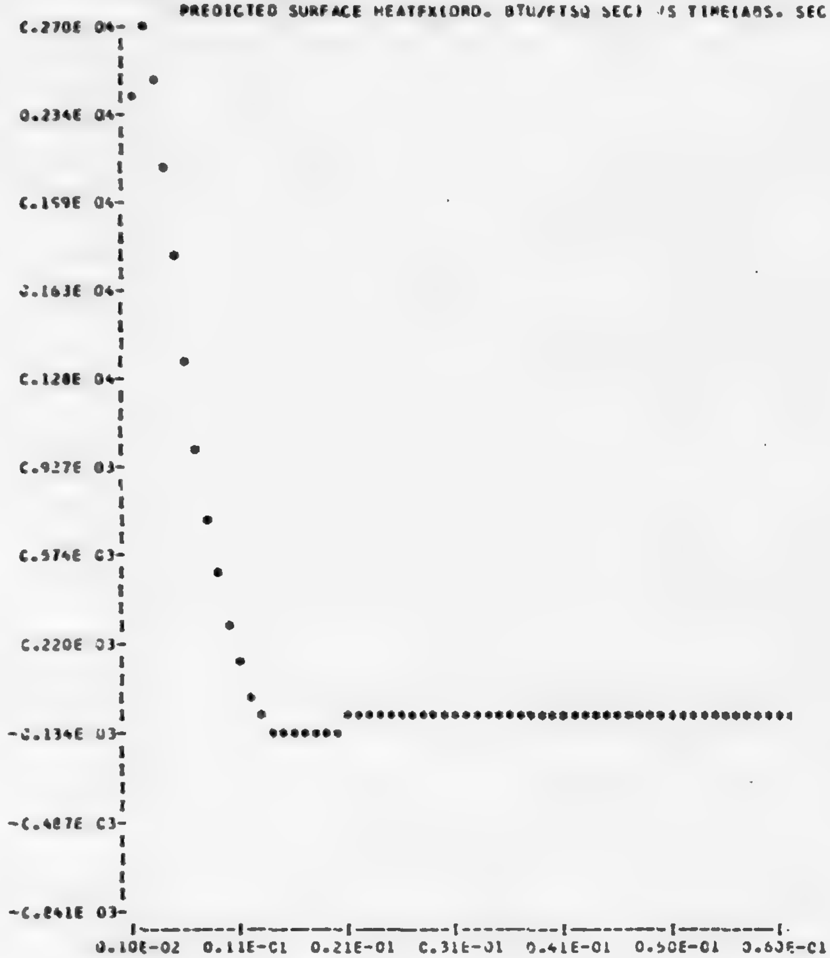
PREDICTED SURFACE HEATFX(URD. BTU/FTSQ SEC) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.12354E 04
MIN. X = 0.10000E-02 MIN. Y = -0.31802E 04

SCALE CN X-AXIS = 0.99 0E-02 UNITS/INCH
SCALE CN Y-AXIS = 0.52 7E 03 UNITS/INCH

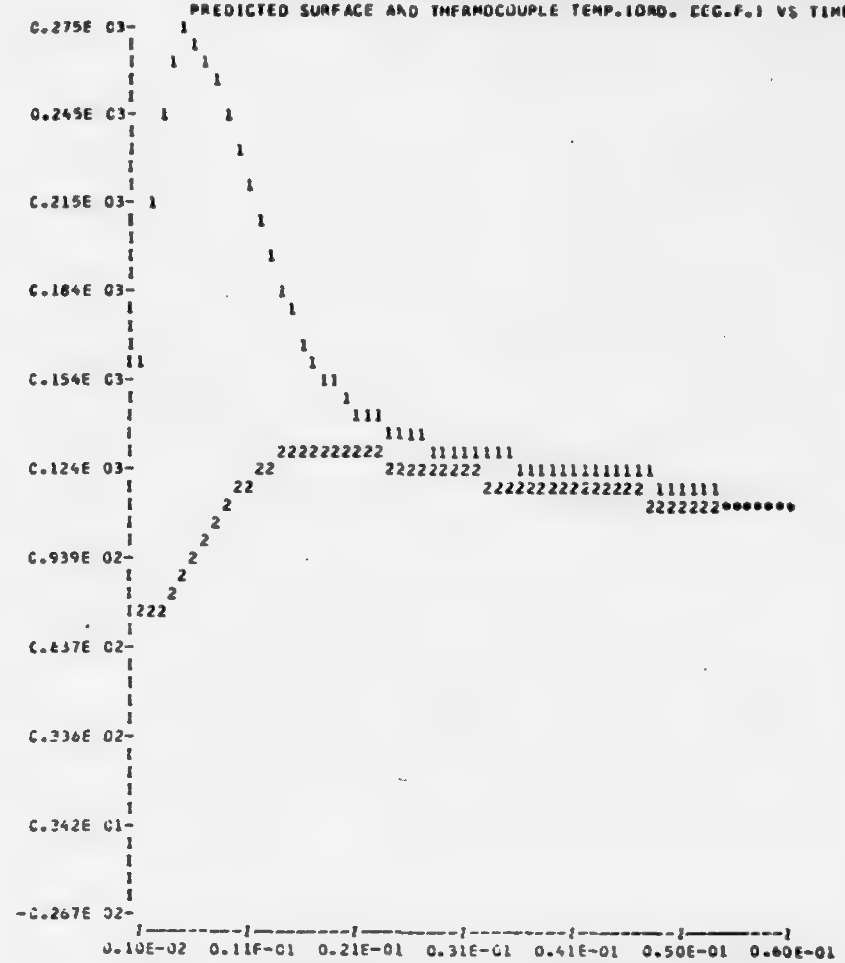
Figure 7 M60 GUN THERMOCOUPLE L 9.0 INCHES FROM BREECH
PREDICTED SURFACE HEATFX(ORD. BTU/FTSQ SEC) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.26952E C4
MIN. X = 0.10000E-02 MIN. Y = -0.94083E 03

SCALE CN X-AXIS = 0.99000E-02 UNITS/INCH
SCALE CN Y-AXIS = 0.42433E 03 UNITS/INCH

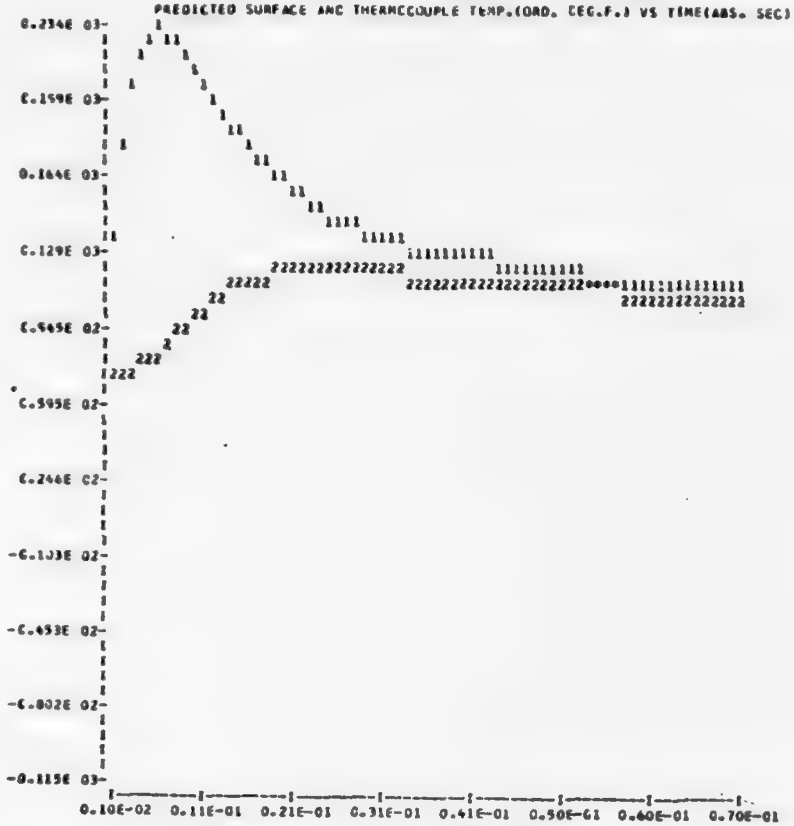
Figure 8 M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH
PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.27482E 03
MIN. X = 0.10000E-02 MIN. Y = -0.26731E 02

SCALE CN X-AXIS = 0.99000E-02 UNITS/INCH
SCALE CN Y-AXIS = 0.36186E 02 UNITS/INCH

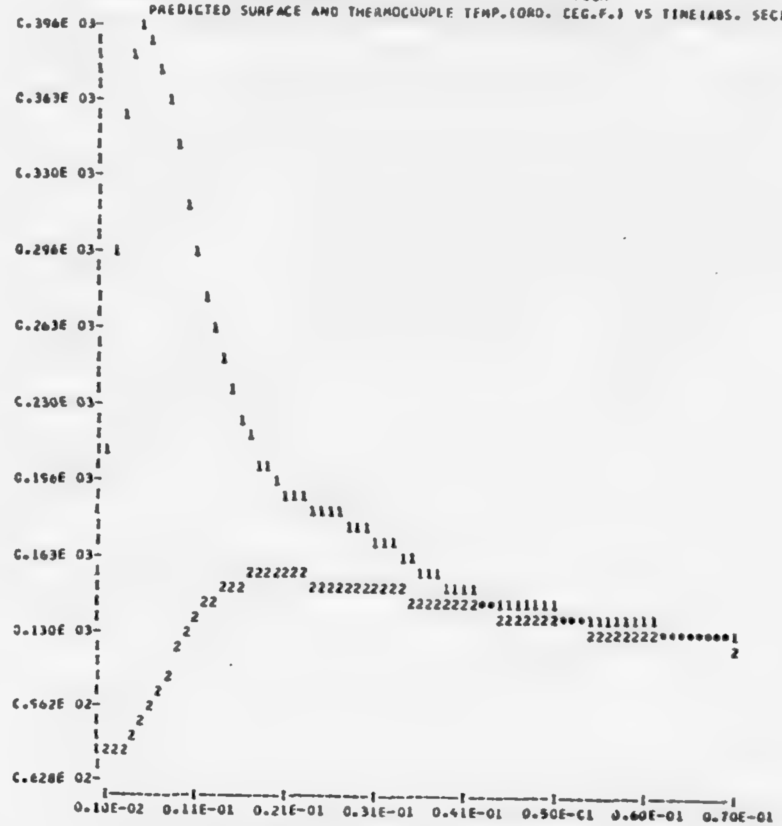
Figure 9 M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH
PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.23425E 03
MIN. X = 0.10000E-02 MIN. Y = -0.11517E 03

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.41130E 02 UNITS/INCH

Figure 10 M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH
PREDICTED SURFACE AND THERMOCOUPLE TEMP.(ORD. DEG.F.) VS TIME(ABS. SEC)



MAX. X = 0.10000E 00 MAX. Y = 0.39641E 03
MIN. X = 0.10000E-02 MIN. Y = 0.62848E 02

SCALE ON X-AXIS = 0.99000E-02 UNITS/INCH
SCALE ON Y-AXIS = 0.40028E 02 UNITS/INCH

V. CONCLUSION AND SUGGESTION

The new inversion computer program was thoroughly tested for its applicability and accuracy. The program can invert an intrinsic temperature response to predict the case of a constant surface heat flux or the case of a periodic surface temperature within 2% of deviation except at the extremely short time period. This was achieved with the maximum number of 20 input data points.

It is expected that the accuracy of the prediction will increase if the number of data input is increased.

Based on the experience gained in working with the program the following suggestions are thought relevant.

(1) In conducting an experiment it is vital that both temperature and the time in the intrinsic measurement of surface heat flux and temperature must be much more accurate than the direct measurement. This is because the inversion problem always involves predicting a large heat flux or temperature variation from the data with small variation. Therefore a slight error in the time or temperature measurement a large error will result in the prediction of the surface heat flux or temperature. For example, in the prediction of gun barrel heat flux considered in Section III an error of two milliseconds in time will lead to 100 percent error in prediction of the surface heat flux.

(2) In selecting the data points for input to the computer program care must be excericised not to create a locally abrupt jump in the data. An abrupt change in data points will often introduce an abnormal fitting of a curve in its neighborhood and hence resulting in an incorrect prediction of the surface heat flux and temperature. If indeed the abrupt jump of the data must be used, then more data points in its neighborhood must also be chosen.

REFERENCES

1. Stolz, G., Jr. "Numerical Solutions to an Inverse Problem of Heat Conduction for Simple Shapes", Journal of Heat Transfer, Vol. 82, 1960, p. 20-26.

2. Beck, J.V., "Calculation of Surface Heat Flux From an Internal Temperature History", ASME Paper 52-HT-46, also, Nuclear Engineering Design, Vol. 7, 1968, p. 170-178.

3. Williams, S. D. and Curry, D. M. "Determination of Surface Heat Flux Using a Single Embedded Thermocouple", NASA TM X-58176 Johnson Space Center, Houston, Texas, Feb. 1976.

4. Carslaw, H. D. and Jaeger, J. C., "Conduction of Heat in Solids", Oxford University Press, London, 1959, p. 112.

5. Burggraf, O. R., "An Exact Solution of the Inverse Problem in Heat Conduction Theory and Application", Journal of Heat Transfer, Vol. 86, 1964, p. 373-382.

6. Koveryanov, V. A., "Inverse Problem of Non-Steady Thermal Conductivity", Teplofizika Vysokiph Temperature, Vol. 5, No. 1, 1967, p. 141-143.

7. Shumakov, N. V., "A Method for the Experimental Study of the Process of Heating a Solid Body", Journal of Technical Physics of the Academy of Sciences USSR, Vol. 2, 1957, p. 771-777.

8. Sparrow, E. M., Hadji-Sheikh, A., and Lundgren, T. S., "The Inverse Problem in Transient Heat Conduction", Journal of Applied Mechanics, Vol. 86, 1964, p. 369-375.

9. Imber, M. and Khan, J. "Prediction of Transient Temperature Distributions with Embedded Thermocouples", Journal of AIAA, Vol. 10, No. 6, 1972, p. 784-789.

10. Imber, M., "A Temperature Extrapolation Method for Hollow Cylinders", Journal of AIAA, Vol. 11, No. 1, 1973, p. 117-118.

11. Sabherwal, K. C., "An Inverse Problem of Transient Heat Conduction", Indian Journal of Pure and Applied Physics, Vol. 3, 1965, p. 397-398.

12. Masket, A. V. and Vastano, A. C. "Interior Problems of Mathematical Physics Part II Heat Conduction", Amerian Journal of Physics, Vol. 30, 1962, p. 796-803.

13. Deverall, L. I., and Channapragada, R. S., "A New Integral Equation for Heat Flux in Inverse Heat Conduction", Journal of Heat Transfer, Vol. 88, 1966, p. 329-328.

14. Chen, C. J., and Thomsen, D. M., "On Transient Cylindrical Surface Heat Flux Prediction from Interior Temperature Response", AIAA Journal, Vol. 13, No. 5, 1975, p. 697-699.

15. Chen, C. J., and Chiou, J. S., "An Investigation of a Remote Transient Heat Flux Sensor" Part I Prediction of Surface Temperature and Heat Flux, Contract Report E-CJC-74-001, August, 1974.

16. Chen, C. J., and Li, P., "Error Analysis of an Intrinsic Transient Heat Flux Sensor", presented at the 16th National Heat Transfer Conference, St. Louis, Missouri, August, 1976.

APPENDIX I. ANALYSIS OF THE INVERSION PROBLEM

Consider a slab, having a sufficient wall thickness, L, such that the outer surface temperature has a negligible response when the inner surface is exposed to a transient heat flux. A probe, for example a thermocouple, is located at $X = X_1$ and it is normally desirable, as reported by Chen and Li [16], to be close to the heating surface since a better transient response and more accurate experimental measurements can be obtained to reduce error amplification in the mathematical inversion program. Under these circumstances we thus assume in the analysis $L/X_1 >> 1$. The governing equation for the transient heat conduction may be written in a dimensionless form as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \tag{1}$$

with the initial and boundary conditions

$$\theta(x, o) = 0 \tag{2}$$

$$\theta(\infty, t) = 0 \tag{3}$$

$$\theta(1, t) = f(t) \tag{4}$$

Where $x = X/X_1$ is a dimensionless distance from the heating surface and $t = \alpha\tau/X_1^2$ is a dimensionless time or Fourier number with α being the thermal diffusivity and τ the real time. $\theta = (T - T_o)/T_o$ is the dimensionless temperature above the initially uniform temperature T_o. $f(t) = (F(\tau) - T_o)/T_o$ is the dimensionless measured temperature response at $x = 1$ with $F(\tau)$ being the measured absolute temperature. The

inversion problem is then given the interior temperature $f(t)$ to predict the surface temperature $\theta(0,t)$ and the surface heat flux per unit area $q(0,t)$ or $\left.\frac{\partial\theta}{\partial x}\right|_{o} = -q(0,t)X_1/T_o\kappa$. Here κ is the thermal conductivity of the solid.

The above problem may be solved by Laplace Transformation. Let the transformation be:

$$\bar{\theta}(x,s) = \int_0^{\infty} \theta(x,t)e^{-ts}\,dt \tag{5}$$

where θ is continuous otherwise satisfied the Dirichlet's condition. The temperature function θ is recovered by inversion of the Laplace Transformation as:

$$\theta(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{\theta}e^{st}\,ds \tag{6}$$

where c is a suitable positive value. Equation (1) and the initial condition (2) under transformation (5) become:

$$\frac{d^2\bar{\theta}}{dx^2} = s\bar{\theta} \tag{7}$$

which has a solution, with integration constants A and B,

$$\bar{\theta}(x,s) = A\,e^{\sqrt{s}x} + Be^{-\sqrt{s}x} \tag{8}$$

The transformation of boundary conditions (3) and (4) into the Laplace

plane give $\bar{\theta}(\infty,s) = 0$, $\bar{\theta}(1,s) = \bar{f}(s)$. Substitution of the boundary conditions into equation (8), we get $\bar{\theta}$ and its derivative as:

$$\bar{\theta}(x,s) = \bar{f}(s)\, e^{\sqrt{s}(1 - x)} \tag{9}$$

$$-\frac{\partial\bar{\theta}}{\partial x} = \sqrt{s}\ \bar{f}(s)\, e^{\sqrt{s}(1 - x)} \tag{10}$$

According to Chen and Thomsen [14], we choose the temperature response as measured by a probe to be represented by a polynomial

$$f(t) = \sum_{n=1}^{N} b_n\, (4t)^n\, \Gamma(n+1)\, i^{2n}\, \mathrm{erfc}\, \left(\frac{1}{2\sqrt{t}}\right) \tag{11}$$

or in Laplace plane

$$\bar{f}(s) = e^{-\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\, \frac{b_n}{s^{1+n}} \tag{12}$$

The b_n's are coefficients of the expansion to be determined such that the N term polynomial describes the temperature response $f(t)$ measured at $x = 1$. With equation (12), equations (9) and (10) can be simplified to

$$\bar{\theta}(x,s) = e^{-x\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\, \frac{b_n}{s^{1+n}} \tag{13}$$

$$-\frac{\partial\bar{\theta}}{\partial x} = e^{-x\sqrt{s}} \sum_{n=1}^{N} \Gamma(n + 1)\, \frac{b_n}{s^{1/2+n}} \tag{14}$$

The inversion of equations (13) and (14) at $x = 0$ give:

$$\theta(0,t) = \sum_{n=1}^{N} b_n \, t^n \tag{15}$$

$$-\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0} = \sum_{n=1}^{N} b_n \, t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \tag{16}$$

where $\theta(0,t)$ gives the surface temperature and $-\frac{\partial\theta(0,t)}{\partial x}$ gives the surface heat flux $\frac{qX_1}{\kappa T_o}$ as a function of time.

The integral of the error function in (11) is defined as:

$$i^{2n} \operatorname{erfc}\left(\frac{1}{2\sqrt{t}}\right) = \int_{\left(\frac{1}{2\sqrt{t}}\right)}^{\infty} i^{2n-1} \operatorname{erfc}(y)\, dy \tag{17}$$

with $n = 0$, $\operatorname{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_y^{\infty} e^{-x^2} dx$. $\Gamma(n)$ in equations (11) and (16) is the gamma function or Euler's integral function of the second kind.

$$\Gamma(n) = \int_0^{\infty} e^{-\omega} \omega^{n-1} \, d\omega \tag{18}$$

It should be remarked that the choice of the particular form (11) is to ensure the convergence of the solution on the Laplace plane and an analytic inversion back to the physical plane. With b_n coefficients determined from equation (11) and the experimental measurement of the temperature response $f(t)$ at $x = 1$, the surface temperature, $T_w(t)$, is obtained from equation (15) as

$$T_w(t) = T_o \left(1 + \sum_{n=1}^{N} b_n \, t^n\right) \tag{19}$$

and the heat flux, $q(t) = \frac{-T_o \kappa}{X_1} \left.\frac{\partial\theta}{\partial x}\right|_{x=0}$, from equation (16) as:

$$q(t) = \frac{-T_o \kappa}{X_1} \sum_{n=1}^{N} b_n \, t^{n-1/2} \frac{\Gamma(n+1)}{\Gamma(n+1/2)} \tag{20}$$

The above solution is the exact solution for predicting the transient surface heat flux and temperature valid for both short and long time durations as long as the slab is thick enough such that the outer surface maintains its initial temperature. The feature of the present solution is the polynomial (11) which on the Laplace plane gives a term in equation (12) $\exp(-\sqrt{s})$ to cancel the term $\exp(\sqrt{s})$ in equations (9) and (10). This polynomial (11) as suggested by Chen and Thomsen [15] makes the present solution simpler than many inversion solutions derived in the past and valid in any time.

If the fluid temperature, $T_g(t)$, away from the surface of the slab is known, then the instantaneous heat transfer coefficients, $h(t)$, can be determined from Newton's cooling law as:

$$h(t) = \frac{q(t)}{T_g(t) - T_w(t)} \tag{21}$$

where heat flux, $q(t)$, and wall temperature $T_w(t)$ are given by equations (19) and (20), and the average heat transfer coefficient up to time t, $\bar{h}(t)$, can be defined as:

$$\bar{h}(t) = \frac{\int_0^t q(t')\,dt'}{\int_0^t [T_g(t') - T_w(t')]\,dt'} \tag{22}$$

APPENDIX II

IMPROVED COMPUTER PROGRAM

Cartesian Inversion Problem

```
FORTRAN IV G LEVEL  21              MAIN              DATE = 76157         15/33/00               PAGE 0001

              C I      ** DATA INPUT **                                                  I
              C I      ****************                                                  I
              C I                                                                        I
              C I           TITLE OF DATA(BLANCK FIRST 20 SPACE)          FIRST CARD     I
              C I           BORRAD=BORE RADIUS OF CYLINDER (FT.)          SECOND CARD    I
              C I           OUTR= OUTER RADIUS (FT.)                      THIRD CARD     I
              C I           THICK=OUTER RADIUS-BORRED (FT.)                              I
              C I           DIS=BORE TO THERMOCOUPLE DISTANCE (FT.)       FOURTH CARD    I
              C I           TEMPO=INITIAL THERMOCOUPLE TEMPERATURE(F.)    FIFTH  CARD    I
              C I           TG(1)= INITAL GAS CORE TEMPERATURE AT NORM TIME 0.00001      I
              C I                                                         SIXTH CARD     I
              C I           ALP=THERMAL DIFFUSIVITY OF MATERIAL(FTSC/SEC)SEVENTH CARD    I
              C I           THCON=THERMAL CONDUCTIVITY OF MATERIAL(BTU/FT.SEC.DEG.F)     I
              C I                                                         EIGHTH CARD    I
              C I           NP=NUMBER OF TIME-TEMPERATURE DATA  PAIRS UP TO 25 PAIRS     I
              C I                                                         NINTH CARD     I
              C I           NB= NUMBER OF B COEFF. TO BE FITTED           TENTH CARD     I
              C I           FROM 11TH  CN AR PAIRS OF TIME(I) AND TEMP(I)                I
              C I           TIME(I)=TIME FROM  THERMOCOUPLE RESPONSE CURVE(SEC.)         I
              C I           TEMP(I)= TEMPERATURE FROM THERMOCOUPLE RESPONSE (DEG. F)     I
              C I           TSHFT = TIME SHIFTED FROM DATA READ IN(SEC)  LAST CARD       I
              C I________________________________________________________________________I
              C
              C
              C ________________________________________________________________________
              C I           DECLARATION STATEMENTS   START COMPUTER PROGRAM              I
              C I________________________________________________________________________I
              C
0001                IMPLICIT REAL*8(A-H,O-Z)
0002                DIMENSION         TG(210),QM(210),DTWM(210),HM(210),TGM(210),
                   1TWM(210),TOT(210),TEMO(210),HT(210),EQUP(210),DTEMR(210)
0003                DIMENSION B(26),BERFC(52),TEMP(26),TIME(26),T(26),X(26),GAM(28),
                   1C(26,26),FN(26),          G(26,210),H(26,210),ECUE(210),DTEMP(210)
                   2,TCT(26),TEMC(210),TRAN(26,26),QTQ(26,26)
0004                DATA EPS/.1D-75/
              C
              C ________________________________________________________________________
              C I           READ DATA CARDS INTO PROGRAM                                 I
              C I________________________________________________________________________I
              C
0005                CALL ERRSET(208,0,-1,1,1)
0006          199   FORMAT(     20X,50H
                   1                )
0007          200   FORMAT(F20.10)
0008          201   FORMAT(I2)
0009          1     READ (5,199,END=999)                                      FIRST
0010                READ (5,200) BORRAD                                       SECOND
0011                READ (5,200) OUTR                                         THIRD
0012                READ(5,200) DIS                                           FOURTH
0013                READ(5,200) TEMPO                                         FIFTH
0014                READ (5,200) TG(1)                                        SISTH
0015                READ(5,200) ALP                                           SEVENTH
0016                READ(5,200) THCON                                         EIGHTH
0017            2   READ(5,201, END=999) NP                                   NINTH
0018            3   READ (5,201) NB                                           TENTH
0019                WRITE (6,198)
0020          198   FORMAT (1H1)
0021                WRITE (6,199)
```

```
          WRITE(6,100)
    100   FORMAT ( 20X,' BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM')
          WRITE (6,109) NB
             DC 10 I=1,NP
             READ(5,202) TIME(I),TEMP(I)
     10      CCNTINUE
          WRITE(6,107)
          DO 16 I=1,NP
     16      WRITE(6,108) TIME(I),TEMP(I)
    202      FORMAT(2F20.10)
          REAC (5,200)  TSHFT
          WRITE (6,130) TSHFT
C
C ____________________________________________________________________
C I           NCW THAT THE CATA HAS BEEN INPUT START CALCULATICNS      I
C I      FIRST SETUP CATA NEEDED FOR THE CALL TC PERFC                 I
C I      WHERE PERFC IS INTEGRATED ERROR FUNCTICN                      I
C I____________________________________________________________________I
C
                      DO 5 I=1,NP
      5               TIME(I) = TIME(I) - TSHFT
          THICK=CUTR-BORRAD
          RO=0.5D0
          GI=1.0D0
          TR1=ALP/(DIS**2)
C
C I -------------------------------------------------------------------I
C I           CACULATE DIMENSIONLESS TIME INCREMENT  'TEMIN'           I
C I      THE CONSTANT CRLT VARIES WITH WHAT REAL TIME RANGE WE WANT    I
C I      BUT NOT DEPENDS CN DIFFERENT MATERIAL OR THERMOCOUPLE         I
C I      LOCATION                                                      I
C I -------------------------------------------------------------------I
C
          CRLT=0.1D-2
          TEMIN=TR1*CRLT
          XXI=0.5D0*GI
          WRITE (6,203) GI,TR1
    203   FORMAT ('-FOR CHECKING USE  GI=',1F7.4 ,'  TR1=',1F7.4 )
          DO 20 I=1,NP
             T(I)=TIME(I)*TR1
     20   X(I)=0.5D0/DSQRT(T(I))
     21   CONTINUE
C
C
C ____________________________________________________________________
C I      USE THE SUBROUTINE PERFC TO CALCULATE INTEGRATED ERROR        I
C I      FUNCTICN AND SET UP  C(I,J) MATRIX FOR THE SOLUTION OF B      I
C I____________________________________________________________________I
C
          NN1 =NB+1
    204   FORMAT ('-NN1 = ', I2)
          WRITE (6,204)  NN1
          DC 30 I=1,NP
     30      C(I,NN1)=(TEMP(I)-TEMPO )/(460.0D0+TEMPO)
          DO 40 I=1,NP
          NB2=NB*2.0D0
          XI = X(I)
          CALL PERFC (NB2,BERFC,XI)
```

```
            DO 40 J=1,NB
         J2=2*J
         C(I,J)=(4.0D0*T(I))**J*DERFC(J2)*DGAMMA(J+1.0D0)
   40    CONTINUE
C
C ----------------------------------------------------------------------
C I      NOW B COEFFICIENT FOR THE THE TEMPERATURE RESPONSE AT          I
C I      THE THERMOCOUPLE ARE SOLVED EITHER BY EXACT OR LEAST SQAURE    I
C I_____________________________________________________________________I
C
         NT=NP+1
         IF (NP - NB) 42, 43, 42
   42    CALL LTSQER (NP,NB,C,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
         GO TO 44
   43    DET = SIMUL (NP, C, B, EPS, 1,11)
   44    CONTINUE
C
C ----------------------------------------------------------------------
C I            NOW CALCULATE HEAT FLUX FUNCTION  AND                    I
C I            CALCULATE TEMPERATURE FUNCTION                           I
C I_____________________________________________________________________I
C
         TEM = TEMIN
         DO 55 I=1,200
            DO 50 J=1,NB
         G(J,I)= TEM**J
         H(J,I)=-TEM**(J-0.5D0)*DGAMMA(J+1.0D0)/DGAMMA(J+0.5D0)
   50          CONTINUE
         TEM = TEM + TEMIN
   55       CONTINUE
C
C
C ----------------------------------------------------------------------
C I            CALCULATE TEMPERATURE AND HEAT FLUX                      I
C I_____________________________________________________________________I
C
         DO 75 I=1,200
            DELT=0.0D0
            QUE=0.0D0
            DO 70 J=1,NB
               DELT=DELT+B(J)*G(J,I)
   70          QUE=QUE+B(J)*H(J,I)
            DTEMP(I)=DELT
         DTEMR(I)=DTEMP(I)*(460.0D0+TEMPO)+TEMPO
            EQUE(I)=-QUE
   75    EQUR(I)= EQUE(I)*(460.0D0+TEMPO)*THCON/DIS
C
C ----------------------------------------------------------------------
C I      CALCULATE INST. AND MEAN QUANTITIES FOR TEMP. AND HEAT T.COFF.I
C I_____________________________________________________________________I
C
            HT(1)=0.0D0
            TGM(1)=0.0D0
            QM(1)=0.0D0
            DTWM(1)=0.0D0
            TWM(1)=0.0D0
            HM(1)=0.0D0
            SUMTW=0.0D0
```

56

```
FORTRAN IV G LEVEL  21                MAIN              DATE = 76157        15/33/00              PAGE 0005

0140         106 FORMAT(' NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=',I2)
0141             WRITE (6,105) NP
0142             WRITE(6,107)
0143             DO 15 I=1,NP
0144         15     WRITE(6,108) TIME(I),TEMP(I)
0145             WRITE (6,130) TSHFT
0146          109 FORMAT (' NUMBER OF B(I) COFF. TO BE FITTED = ',I2)
0147         130 FORMAT(' TIME OF DATA SHIFTED BY(SEC)' ,F10.6)
0148          108    FORMAT(' ',7X,F20.10,6X,F20.10)
0149          107 FORMAT('-',13X,'TIME',5X,'TEMPERATURE')
0150             DO 41 I=1,AB
0151          229 FORMAT (' COEFFICIENTS OF B(I)=', D16.8, '      I =',I2)
0152          41 WRITE (6,229) B(I),I
0153             WRITE (6,198)
0154             WRITE (6,199)
0155             WRITE(6,113)
0156         113 FORMAT (30X,'  TEMPERATURE AND HEAT FLUX '/,30X,'  PREDICTED BY',
            1  '  JENQ-SHING CHIOU ',///,2X,
            2'(NORTM) (REALTM) (NOR.FLUX) (REAL.FLUX) (NOR.T)   (REAL.SUF.T) (
            3TM.CO.T)')
0157             WRITE (6,115)
0158          115 FORMAT(' ','(OMLESS) (SEC.)      (OMLESS) (BTU/FTSQ SEC) (OMLESS)
            1(DEG.F.) (DEG.F.)',/)
0159             TEM = TEMIN
0160             TINC=TEM
0161             TRINC= TEMIN/TR1
0162             TREM=TRINC
0163             DO 90 I=1,200
0164                WRITE (6,114) TEM,TREM,EQUE(I),EQUR(I),DTEMP(I),DTEMP(I),TEMC
            1(I)
0165          114 FORMAT (F9.6,F11.7,2(F10.4),2(F12.4),F15.8)
0166                TEM=TEM+TINC
0167                TREM=TREM+TRINC
0168          90    CONTINUE
0169             WRITE(6,113)
0170             WRITE (6,115)
0171             WRITE (6,199)
0172             GO TO 1
            C
            C
            C ______________________________________________________________________
            C I   THIS IS THE END OF THE MAIN PROGRAM                               I
            C I_____________________________________________________________________I
            C
0173          999 WRITE(6,1000)
0174         1000 FORMAT('-ALL DATA PROCESSED-EXECUTION OF EXECUTION')
0175             DEBUG SUBCHK
0176             END
```

```
0053                SUMTG=0.0D0
0054                SUMQ    =0.0D0
0055                SUMDT=0.0D0
0056            TG(1)=(TG(1)-TEMPO)/(460.0D0+TEMPO)
0057            DO 80 I=1,200
0058                F=I
0059                SUMQ    = SUMQ    +EQUE(I)
0100                CM(I) = SUMQ/F
0101                SUMTG= SUMTG + TG(I)
0102                TGM(I)= SUMTG/F
0103                SUMDT = SUMDT + TG(I)- DTEMP(I)
0104                SUMTW = SUMTW + DTEMP(I)
0105                TWM(I)=SUMTW/F
0106                HM(I)= SUMQ   /SUMDT
0107                HT(I)=EQUE(I)/( TG(I) - DTEMP(I))
0108            TG(I+1)=(DTEMP(I)+DTEMP(I+1))/2.0D0+(EQUE(I)+EQUE(I+1))/(2.0D0*
              1         HT(I))
0109       80       CONTINUE
          C
          C ________________________________________________________________
          C I           CALCULATE THERMOCOUPLE TEMPERATURE FROM FITTED EQ.     I
          C I______________________________________________________________I
          C
0110            TEM = TEMIN
0111            DO 78 I=1,200
0112            TEMC(I) = 0.0D0
0113            XX=0.5D0/DSQRT(TEM)
0114            CALL PERFC (NB2,BERFC,XX)
0115                DO 76 J=1,NB
0116                J2=J*2.0D0
0117            TCT(J)=(TEM*4.0D0)**J*BERFC(J2)*DGAMMA(J+1.0D0)
0118            TEMC(I) = TEMC(I) + B(J)*TCT(J)
0119       76       CONTINUE
0120            TEM = TEM + TEMIN
0121            TEMC(I)=TEMC(I)*(TEMPO+460.0D0)+TEMPO
0122       78   CONTINUE
          C
          C ________________________________________________________________
          C I           NOW OUTPUT THE DATA                                  I
          C I______________________________________________________________I
          C
0123            WRITE (6,198)
0124            WRITE (6,199)
0125            WRITE(6,101) BORRAD
0126      101   FORMAT('-BORE RADIUS  (FT.) =',F10.5)
0127            WRITE (6,119) OUTR
0128      119   FORMAT  (' OUTER RADIUS (FT.)=',F10.5)
0129            WRITE(6,102) DIS
0130      102   FORMAT(' BORE TO THERMOCOUPLE  DISTANCE (FT)=',F10.6)
0131            WRITE(6,103) TEMPO
0132      103   FORMAT(' INITIAL THERMOCOUPLE TEMPERATURE (F.)=',F10.4)
0133            WRITE (6,301) TG(1)
0134      301   FORMAT (' INITIAL GAS TEMPERATURE (F.)=',F10.4)
0135            WRITE(6,104) ALP
0136      104   FORMAT (' THERMAL DIFFUSIVITY  (FTSQ/SEC) =',F15.8)
0137            WRITE(6,105) THCON
0138      105   FORMAT(' THERMAL  CONDUCTIVITY(BTU/FT.SEC.F.)=',F15.8)
0139            WRITE(6,106) NP
```

```
FCRTRAN IV G LEVEL  21                PERFC              DATE = 76157          15/33/00                PAGE 0001

0001                SUBROUTINE PERFC(NP,BERFC,X)
             C
             C
             C ____________________________________________________________________________
             C I               THIS SUBROUTINE CALCULATES THE REPEATED INTERGRALS          I
             C I         OF THE COMPLEMENTARY ERROR FUNCTICN                               I
             C I                                                                           I
             C I         NP= NUM.ER OF REPEATS INTEREALS TO BE CALCULATED                  I
             C I         CERFC= REAL*8 ARRAY FOR ERROR F.                                  I
             C I         X= THE INITIAL VALUF FOR ERROR FUNCTICN                           I
             C I                                                                           I
             C I___________________________________________________________________________I
             C
             C
0002                IMPLICIT REAL*8(A-H,O-Z)
0003                DIMENSION BERFC(52)
0004                DATA SRIPI/ 0.5 418958355/
             C
             C ____________________________________________________________________________
             C I               INITIALIZE & CALCULATE I(1)ERFC &I(2)ERFC DEPENDING          I
             C I               CN NP                                                        I
             C I___________________________________________________________________________I
             C
0005                CALL ERRSET(208,0,-1,1,1)
0006                DO 11 I =1,40
0007                BERFC(I) =0.000
0008           11   CONTINJE
0009                X2=X*X
0010                BERFC1=SRIPI*DEXP(-X2)-X*DERFC(X)
0011                BERFC(1)=BERFC1
0012                IF(NP.EQ.1) RETURN
0013                BERFC2=0.25D0*((1.0D0+2.0D0*X2)*DERFC(X)-2.0D0*SRIPI*X*DEXP(-X2))
0014                BERFC(2)=BERFC2
0015                IF(NP.EQ.2)  RETURN
             C
             C ____________________________________________________________________________
             C I               NOW GO INTO DO LOOP & CALCULATE I(NP)ERFC                    I
             C I___________________________________________________________________________I
             C
0016                DO 10 I=3,NP
0017                BERFC(I)=0.5D0/I*(BERFC1-2.0D0*X*BERFC2)
0018                     BERFC1=BERFC2
0019          10         BERFC2=BERFC(I)
0020                DEBUG SUBCHK
0021                RETURN
0022                END
```

```
0001           SUBROUTINE  LTSQER (M,JJ,PHI,B,EPS,TIME,NT,TRAN,QTQ,TEMPO)
         C
         C ______________________________________________________________
         C I   THIS PROGRAM CALCULATES THE LEAST EQUARES COEFFICIENTS FOR      I
         C I   MODIFIED POLYNOMIAL WHICH IS POLYNOMIAL TIMES THE INTEGRATED    I
         C I   ERROR FUCTICN                                                   I
         C I                                                                   I
         C I   M IS THE NUMBER OF CATA PAIRS TO BE USED                        I
         C I   JJ IS THE NUMBER OF DEGREE TO BE FITTED                         I
         C I_______________________________________________________________I
         C
         C
0002           REAL*8 PHI(26,26),FX(26),TRAN(26,26),QTQ(26,26),X(26),DET, FPX,
              1B(26),TIME(26)
0003           CALL ERRSET(208,0,-1,1,1)
0004           LDIM=50
0005           N1=JJ+1
0006           DO 1001 I=1,M
0007           X(I)=TIME(I)
0008           FX(I)=PHI(I,N1)
0009      1001 CONTINUE
0010           ERROR =0.000
         C
         C ______________________________________________________________
         C I   FIND THE TRANSPOSE CF THE PHI    MATRIX FOR LSTSQ F TTING AND   I
         C I   PERFORM THE MULTIPLICATION   AND GET JJ BY JJ MATRIX            I
         C I_______________________________________________________________I
         C
0011           CALL TRANS (PHI, TRAN,M,JJ,NT)
0012           CALL MULT (PHI,TRAN,JJ,M,QTQ,NT)
0013           DO 4 J=1,JJ
0014           QTQ(J,N1)=0.000
0015           DO 4 K=1,M
0016       4   QTQ(J,N1)=QTQ(J,N1) + TRAN(J,K)*FX(K)
0017           WRITE (6,5) (I,QTQ(I,N1),I=1,JJ)
0018       5   FORMAT (' I=',I2,'   QTQ(I,N1)=',D16.7)
         C
         C ______________________________________________________________
         C I   FIND THE SOLUTICN TC THE NORMAL EQUATIONS                       I
         C I_______________________________________________________________I
         C
0019           DET = SIMUL (JJ,QTQ,B,EPS,1,20)
0020           WRITE (6,100)
0021     100   FORMAT ('  LEAST SCUARE COEFFICIENTS ')
0022           WRITE (6,6) (I,B(I)   ,I=1,JJ)
0023       6   FORMAT (' I= ',I2,  '  B(I)= ', D16.7)
0024           WRITE (6,9)
0025       9   FORMAT (//' LEAST SQ OUT PUT ',//' KTH TIME',T15,'TEMP. GIVEN ',
              1T30,'FITTED', T42, 'ERRCR',T55,'PER CENTAGE')
0026           DO 15 K=1,M
0027           FPX=TEMPO
0028           DO 8 I=1,JJ
0029       8   FPX=FPX + QTQ(I,N1)* PHI(K,I)
0030           FX(K)= FX(K) + TEMPO
0031           ERR = PHI(K,N1) - FPX + TEMPO
0032           IF (PHI(K,N1).NE. 0.)  PER=ERR/(PHI(K,N1) + TEMPO)*100.0
0033           IF( ABS(ERR).LE.ERRCR) GO TO 15
0034           ERROR = ABS(ERR)
```

```
   15 WRITE (6,17) K,X(K),FX(K), FPX,ERR,PER
   17 FORMAT (1I3,   3(F10.5,5X),F10.6,3X,F6.2)
      WRITE (6,110)   ERROR
  110 FORMAT ('  MAXIMUN ERROR =' ,D16.7)
      DEBUG SUBCHK
      RETURN
      END

      SUBROUTINE TRANS (PH1,TRAN,M,N,N1)
C
C ________________________________________________________________
C I      THIS SUBROUTINE GIVE THE TRANSPOSE OF THE MATRIX
C I_______________________________________________________________I
      REAL*8 PH1(26,26),TRAN(26,26)
      CALL ERRSET(208,0,-1,1,1)
      DO 10 J=1,N
      DO 10 I=1,M
   10 TRAN(J,I)=PH1(I,J)
      RETURN
      END


      SUBROUTINE MULT (PH1,TRAN,N,M,QTQ,N1)
C
C ________________________________________________________________
C I      THIS SUBROUTINE GIVES MULTIPLICATION OF TWO MATRIES       I
C I_______________________________________________________________I
      REAL*8 PH1(26,26),TRAN(26,26),QTQ(26,26)
      CALL ERRSET(208,0,-1,1,1)
      DO 1 I=1,N
      DO 1 J=1,N
      QTQ(I,J)=0.0D0
      DO 1 K=1,M
    1 QTQ(I,J)=QTQ(I,J)+TRAN(I,K)*PH1(K,J)
      RETURN
      END
```

```
      FUNCTION SIMUL(N,A,X,EPS,INDIC,NRC)
C
C ______________________________________________________________
C I   .....GAUSS JORDAN TECHNIQUE MAXIMUM  PIVOT                 I
C I   ....PAGE 290 'APPLIED NUMERICAL METHODS' CARNAHAN          I
C I______________________________________________________________I
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION IROW(60),JCOL(60),JORD(60),Y(60),A(26,26  ),X(26)
      CALL ERRSET(208,0,-1,1,1)
      MAX=N
      IF(INDIC.GE.0) MAX=N+1
C     .....IS N LARGER THAN 50
      IF(N.LE.50) GO TO 5
      WRITE (6,200)
      SIMUL=0.0D0
      RETURN
C     BEGIN ELIMINATION PROCEEDURE
5     DETER=1.0D0
      DO 18 K=1,N
      KM1=K-1
C    .....SEARCH FO R THE PIVOT ELEMENT
      PIVOT=0.0D0
      DO 11 I=1,N
      DO 11 J=1,N
C    ....SCAN IROW AND JCOL ARRAYS FOR INVALID PIVOT SUBSCRIPTS
      IF(K.EQ.1) GO TO 9
      DO 8 ISCAN=1,KM1
      DO 8 JSCAN=1,KM1
      IF(I.EQ.IROW(ISCAN)) GO TO 11
      IF(J.EQ.JCOL(JSCAN)) GO TO 11
8     CONTINUE
9     IF(DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11
      PIVOT=A(I,J)
      IROW(K)=I
      JCOL(K)=J
11     CONTINUE
C     ....INSURE THAT SELECTED PIVOT IS LARGER THAN EPS
      IF(DABS(PIVOT).GT.EPS) GO TO 13
      SIMUL=0.0D0
      RETURN
C     UPDATE THE DETERMINANT VALUE...
13    IROWK=IROW(K)
      JCOLK=JCOL(K)
      DETER=DETER*PIVOT
C     ....NORMALIZE PIVOT ROW ELEMENTS
      DO 14 J=1,MAX
14     A(IROWK,J)=A(IROWK,J)/PIVOT
C    ...CARRY OUT ELIMATION AND DEVELOPE INVERSE
      A(IROWK,JCOLK)=1./PIVOT
      DO 18 I=1,N
      AIJCK=A(I,JCOLK)
      IF(I.EQ.IROWK) GO TO 18
      A(I,JCOLK)=-AIJCK/PIVOT
      DO 17 J=1,MAX
17    IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
18      CONTINUE
C   ....ORDER SOLUTION VALUES AND CREATE JORD ARRAY
```

```
      DO 20 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
      JORD(IROWI)=JCOLI
20    IF(INDIC.GE.0) X(JCOLI)=A(IROWI,MAX)
C     ADJUST SIGN OF DETERMINANT
      INTCH=0
      NM1=N-1
      DO 22 I=1,NM1
      IP1=I+1
      DO 22 J=IP1,N
      IF(JORD(J).GE.JORD(I))  GO TO 22
      JTEMP=JORD(J)
      JORD(J)=JORD(I)
      JORD(I)=JTEMP
      INTCH=INTCH+1
22    CONTINUE
      IF(INTCH/2*2.NE.INTCH) DETER=-DETER
C    ....IF INDIC IS POSITIVE RETURN WITH RESULTX
      IF(INDIC.LE.0) GO TO 26
      SIMUL=DETER
      RETURN
C    .....IF INDIC IS NEGATIVE OR ZERO, UNSCRAMBLE YHE INVERSE
C     ...FIRST BY ROWS
26    DO 28 J=1,N
      DO 27 I=1,N
      IROWI=IROW(I)
      JCOLI=JCOL(I)
27    Y(JCOLI)=A(IROWI,J)
      DO 28  I=1,N
28    A(I,J)=Y(I)
C     ...THEN BY COLUMNS
      DO 30 I=1,N
      DO 29 J=1,N
      IROWJ=IROW(J)
      JCOLJ=JCOL(J)
29    Y(IROWJ)=A(I,JCOLJ)
      DO 30 J=1,N
30    A(I,J)=Y(J)
C    .....RETURN FOR INDIC NEGATIVE OR ZERO
      SIMUL=DETER
      RETURN
C    ....FORMAT OFR OUTPUT STATEMENT
200   FORMAT(10HON TOO BIG)
      DEBUG SUBCHK
      END
```

APPENDIX III M6C DATA

M60 GUN THERMOCOUPLE 7 15.0 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	110.6000000000
0.0200000000	124.5000000000
0.0300000000	123.1000000000
0.0400000000	119.6000000000
0.0500000000	116.9000000000
0.0600000000	114.1000000000
0.0700000000	112.5000000000
0.0800000000	110.3000000000
0.0900000000	107.6000000000
0.1000000000	106.8000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.04417
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001830
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

M60 GUN THERMOCOUPLE 10 21 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	126.2000000000
0.0200000000	131.6000000000
0.0300000000	124.2000000000
0.0400000000	119.6000000000
0.0500000000	116.8000000000
0.0600000000	113.3000000000
0.0700000000	109.8000000000
0.0800000000	107.1000000000
0.0900000000	105.7000000000
0.1000000000	104.2000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.03562
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001670
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

M60 GUN THERMOCOUPLE 4 9.0 INCHES FROM BREECH
BORE SURFACE TEMPERATURE AND HEAT FLUX PROGRAM
NUMBER OF B(I) COFF. TO BE FITTED = 10

TIME	TEMPERATURE
0.0100000000	143.2000000000
0.0200000000	157.3000000000
0.0300000000	151.0000000000
0.0400000000	144.4000000000
0.0500000000	137.5000000000
0.0600000000	132.8000000000
0.0700000000	129.0000000000
0.0800000000	125.9000000000
0.0900000000	122.8000000000
0.1000000000	119.6000000000

TIME OF DATA SHIFTED BY(SEC) -0.002000
BORE RADIUS (FT.) = 0.01250
OUTER RADIUS (FT.)= 0.05000
BORE TO THERMOCOUPLE DISTANCE (FT)= 0.001830
INITIAL THERMOCOUPLE TEMPERATURE (F.)= 78.8000
INITIAL GAS TEMPERATURE (F.)= 4.4937
THERMAL DIFFUSIVITY (FTSQ/SEC) = 0.00010307
THERMAL CONDUCTIVITY(BTU/FT.SEC.F.)= 0.00555555
NUMBER OF TIME TEMPERATURE PAIRS (SEC.,F.)=10
NUMBER OF B(I) COFF. TO BE FITTED = 10

APPENDIX IV. THE CASE OF OSCILATORY SURFACE TEMPERATURE

Consider a slab with a sufficient thickness, ℓ, such that when a surface is subjected to a periodic surface temperature variation with a frequency w the other surface is held at the initial temperature T_o. If properties are assumed constant the governing equation for the problem can be written as

$$\frac{\partial \nu}{\partial t} = \alpha \frac{\partial^2 \nu}{\partial x^2} \tag{1}$$

with initial and boundary conditions as

$$\nu(x, o) = 0 \tag{2}$$

$$\nu(\ell, t) = \sin wt \tag{3}$$

$$\nu(o, t) = 0 \tag{4}$$

where $\nu = (T - T_o)/(T_{max} - T_o)$ and α is the thermal diffusivity.

The solution of the problem according to Carslaw and Jaeger [4] can be written as

$$\nu = 2\alpha\pi \sum_{n=1}^{\infty} (-1)(-1)^n n \left[(\alpha n^2\pi^2 \sin wt - w\ell^2 \cos wt) + w\ell^2 e^{-\alpha n^2\pi^2 t/\ell^2}\right]$$

$$\cdot \sin\left(\frac{n\pi x}{\ell}\right) / [\alpha^2 n^4 \pi^4 + w^2 \ell^2] \qquad (\;)$$

Part III PREDICTION OF TRANSIENT SURFACE HEAT FLUX AND TEMPERATURE ON A HOLLOW CYLINDER

I. INTRODUCTION

In the study of transient heat transfer many efforts have been made on the so-called "inverse problem" [1,2] where a surface heat flux and temperature is to be predicted by the measured data at some location interior to a body.

In the previous works [1-6] the solution is represented in either an integral form after some manipulation of the contour integral from the inverse transform, or in a series form after the expansion of the solution for small and large times. Using Laplace transformation Chen and Thomsen [6] introduced a polynomial in terms of the error function to represent the response of thermocouple measurement and the inversion is accomplished for any transient surface heat flux at the inner surface of a cylindrical tube. However, their inversion solution is valid only for a short duration due to the asymptotic expansion of the modified Bessel function in the inverse Laplace transform. In this study an exact solution obtained from the inverse Laplace transform by the convolution method is given for the case of hollow cylinder. The solution is valid for both constant and variable heat flux and for both short and long time duration.

II. ANALYSIS

Consider a long hollow cylinder with sufficient wall thickness such that the outer surface temperature has a negligible response when the inner surface is exposed to a thermal pulse of a transient process. This condition considerably simplifies the theoretical analysis as the outer boundary may be assumed to be infinite, and only one interior probe of the cylinder

is required in the experimental measurement. The material of the cylinder is considered to be homogeneous and isotropic with constant thermal diffusivity, α. Let R_i and R_o be, respectively, the inner and outer surface radii. R_1 the radius of the probe location and t the dimensionless time. If the temperature of the cylinder is initially uniform at T_o, the mathematical problem governing the temperature T, may be written as

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} \qquad 1 < r < r_o = \infty \tag{1}$$

$$\theta\ (r, o) = 0 \tag{2}$$

$$\theta\ (\infty, t) = 0 \tag{3}$$

$$\theta\ (r_1, t) = f(t) \qquad 1 < r_1 < \infty \tag{4}$$

where $\theta = T - T_o$, $r = R/R_i$, $t = \alpha\tau/R_i^2$, and f(t) is the interior temperature response of the thermocouple measured at $r = r_1$ at the dimensionless time t. The problem is to predict the surface temperature θ (1, t) and heat flux per unit area

$$q = -\ (K/R_1)(\partial\theta/\partial r)\Big|_{r = 1} \tag{5}$$

where K is the thermal conductivity.

The problem can be solved by Laplace transformation. Let the transformation be

$$\bar{\theta}(r, s) = \int_o^\infty \theta e^{-ts} dt \tag{6}$$

when θ satisfies the Dirichlet's condition the temperature function θ is recovered by inversion of the Laplace transformation as

$$\theta(r, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{\theta} e^{st} ds \tag{7}$$

where c is a suitable positive value. Equation (1) and (2) under transformation (6) becomes

$$\frac{d^2\bar{\theta}}{dr^2} + \frac{1}{r}\frac{d\bar{\theta}}{dr} = s\bar{\theta}$$

which has a solution of the form

$$\bar{\theta} = AI_o(pr) + BK_o(pr) \tag{8}$$

where I_o and K_o are modified Bessel functions of the first and second kind with $p = (s)^{1/2}$. With the boundary conditions (3) and (4), (8) becomes

$$\bar{\theta} = \bar{f}(s)\ [K_o(pr)/K_o(pr_1)] \tag{9}$$

where $\bar{f}(s)$ is the Laplace transform of the boundary condition (4).

The temperature response measured at $r = r_1$ can be expresses by a polynomial or numerous other suitable functions. In the present analysis, for reasons to be explained later, f(t) will be represented as

$$f(t) = \sum_{n=1}^{N} b_n \int_0^t F_1(\tau)\ Fn(t - \tau)d\tau \tag{10}$$

If we choose $F_1(t) = \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}}$ and $Fn(t - \tau)$ being any arbitrary function

depending on n, for example $(t - \tau)^n$ etc. then the Laplace transform of Eq. (10) gives

$$\bar{f}(s) = \sum_{n=1}^{N} b_n K_o(pr_1)\bar{F}n(s) \tag{11}$$

where $\bar{F}n(s)$ is the Laplace transform of Fn(t). Substituting Eq. (11) into Eq. (19) we have

$$\bar{\theta}(s, r) = \sum_{n=1}^{N} b_n K_o(pr)\bar{F}n(s) \tag{12}$$

It is noted that the $K_o(pr_1)$ in Eq. (9) has been cancelled by this substitution which explains the choice of $F(t) = \frac{1}{4K} e^{-\frac{r_1^2}{4K}}$ in Eq. (10). The inversion of Eq. (12) gives

$$\theta(t, r) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{r^2}{4\tau}} Fn(t - \tau)d\tau \tag{13}$$

At surface r = 1

$$\theta(t, 1) = \sum_{n=1}^{N} b_n \int_o^t \frac{1}{2\tau} e^{-\frac{1}{4\tau}} Fn(t - \tau)d\tau \tag{14}$$

The temperature gradient and hence heat flux at surface is

$$\left.\frac{\partial\theta}{\partial r}\right|_{r=1} = -\sum_{n=1}^{N} b_n \int_o^t \frac{1}{4\tau^2} e^{-\frac{1}{4\tau}} Fn(t - \tau)d\tau \tag{15}$$

some examples of $Fn(t - \tau)$ frunction and the representation of the thermocouple response are:

Case 1 If $\bar{F}n(s) = \frac{1}{s^2+n^2}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_0^t \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}} \frac{1}{n} \sin n(t - \tau)d\tau \qquad (16)$$

Case 2 If $\bar{F}n(s) = \frac{s}{s^2+n^2}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_0^t \frac{1}{2\tau} e^{-\frac{r_1^2}{4\tau}} \cos n(t - \tau)d\tau \qquad (17)$$

Case 3 If $\bar{F}n(s) = \frac{1}{(s + a)^n}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_0^t \frac{e^{-\frac{r_1^2}{4\tau}} (t - \tau)^{n-1} e^{-a(t-\tau)}}{2\tau(n - 1)} d\tau \qquad (18)$$

Case 4 If $\bar{F}n(s) = s^{-(n+1/2)}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_0^t \frac{e^{-\frac{r_1^2}{4\tau}} 2^n (t - \tau)^{n-1/2}}{2\tau \; 1.3.5...(2n - 1)\sqrt{\pi}} d\tau \qquad (19)$$

Case 5 If $\bar{F}n(s) = \frac{1}{s^n}$

$$\text{then } f(t) = \sum_{n=1}^{N} b_n \int_0^t e^{-\frac{r_1^2}{4\tau}} \frac{(t - \tau)^{n-1}}{2\tau(n - 1)!} d\tau \qquad (20)$$

REFERENCES

[1] Stolz, G. Jr., "Numerical Solutions to an Inverse Problem of Heat Conduction for Simple Shapes", Journal of Heat Transfer, Vol. 82, 1960, p. 20-26.

[2] Sparrow, E. M. Haji-Sheikh, A., and Lundgren, T. S., "The Inverse Problem in Transient Heat Conduction", Journal of Applied Mechanics, Vol. 86, 1964, p. 369-375.

[3] Beck, J. V., "Calculation of Surface Heat Flux From an Internal Temperature History", Nuclear Engineering Design, Vol. 7, 1968, p. 170-178.

[4] Imber, M., and Khan, J., "Prediction of Transient Temperature Distributions with Embedded Thermocouples", Journal of AIAA, Vol. 10, No. 6, 1972, p. 784-789.

[5] Deverall, L. I. and Channapragada, R. S., "A New Integral Equation for Heat Flux in Inverse Heat Conduction", Journal of Heat Transfer, Vol. 88, 1966, p. 329-328.

[6] Chen, C. J., and Thomsen, D. M., "On Transient Cylindrical Surface Heat Flux Prediction from Interior Temperature Response", AIAA Journal, Vol. 13, No. 5, 1975, p. 697-699.